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ARALASH PARABOLO-GIPERBOLIK TENGLAMA UCHUN CHEGARAVIY MASALA

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Annotatsiya: *Ushbu maqolada tip o‘zgarish chiziqlari parallel bo‘lgan parabolo-giperbolik tenglama uchun chegaraviy masala yechimi o‘rganilgan. Masalaning yechimi Volterraning ikkinchi tur integral tenglamasiga keltirib topilgan.*

Kalit so‘zlar: *Parabolo-giperbolik tenglama, chegaraviy masala, Grin funksiyasi, Dirixle formulasi, Abel tenglamasi, Volterraning ikkinchi tur integral tenglamasi.*

I.KIRISH

1-§. Masalaning qo‘yilishi

Dastlab elliptik-giperbolik tipdagi ikkinchi tartibli aralash tenglamalar o‘rganilgan. Bunday tenglamalar bo‘yicha fundamental tadqiqotlar 1920-yillarda italiyalik matematik F. Trikomi tomonidan olib borilgan va S. Gellerstedt, A. V. Bitsadze, K. I. Babenko, I. L. Karol, F.I. Frankl, M.M.Smirnov, M.S. Salaxiddinov, T.D. Jurayev va boshqalar tomonidan rivojlantirilgan.

Keyin aralash tipdagi tenglama tushunchasi ikki yoki uchta klassik tipdagi tenglamalarning har xil kombinatsiyalarini o‘z ichiga olgan holda sezilarli darajada kengaytirildi. Aralash elliptik-parabolik va parabolik-giperbolik tipdagi tenglamalarni intensiv o‘rganish, bir tomondan, aralash tenglamalarning yangi tiplari hali ham nazariy jihatdan kam o‘rganilganligi bilan bog‘liq, boshqa tomondan ular mexanika, fizika va texnikaning muhim masalalarida keng qo‘llaniladi.

Bizga quyidagicha

$$0=\begin{cases} u_{xx}-u_y, & D_1, \\ u_{xx}-u_{yy}, & D_i,\ i=2,3, \end{cases} \qquad (1.1)$$

aralash tenglama berilgan bo‘lsin. Bu yerda D_1-soha, AB, BB_0, B_0A_0 va A_0A segmentlar mos ravishda $y=0$, $x=1$, $y=1$ va $x=0$ to‘g‘ri chiziqlar bilan chegaralangan kvadrat $\{0<x<1, 0<y\le 1\}$; D_2-soha, A_0A segment ya’ni y o‘q hamda A va A_0 nuqtalardan chiquvchi $AC: x+y=0$, $A_0C: y-x=1$ ikki to‘g‘ri

chiziqlarning $C\left(-\frac{1}{2},\frac{1}{2}\right)$ nuqtada kesishidan hosil bo'lgan uchburchakli soha. D_3-soha, BB_0 segment hamda B va B_0 nuqtalardan chiquvchi $BE: \ x-y=1, \ B_0E: \ x+y=2$ ikki to'g'ri chiziqlarning $E\left(\frac{3}{2},\frac{1}{2}\right)$ nuqtada kesishidan hosil bo'lgan uchburchakli soha.

D_1, D_2, va D_3 sohalar to'plamini hamda AA_0 va BB_0 ochiq segmentlarni D bilan belgilaymiz.

Masala A. Quyidagi berilgan shartlarni qanoatlantiruvchi $u(x,y)$ funksiyani toping:

1) AA_0 va BB_0 segmentdagi nuqtalardan tashqari D sohaning hamma joyida $u(x,y)$ funksiya (1.1) tenglamaning regulyar yechimi bo'ladi.

2) $u(x,y)\in C(\bar{D}_j)\cap\left[C^1(D_1\cup AA_0\cup BB_0)\cap C^1(D_2\cup AA_0)\cap C^1(D_3\cup BB_0)\right]$, $j=1,2,3$;

3) Shartlarni qanoatlantiradi

$$u_{|AC}=\psi_1(y), \ \ 0\le y\le\frac{1}{2}, \tag{1.2}$$

$$u_{|BE}=\psi_2(y), \ \ 0\le y\le\frac{1}{2}, \tag{1.3}$$

$$u_{|y=0}=\varphi(x), \ \ 0\le x\le 1; \tag{1.4}$$

(1.4) va bular uchun ulashish shartlarini qanoatlantiradi

$$\begin{aligned} &u(-0,y)=\alpha_1(y)u(+0,y)+\gamma_1(y),\\ &u_x(-0,y)=\beta_1(y)u_x(+0,y)+\delta_1(y),\\ &u(1+0,y)=\alpha_2(y)u(1-0,y)+\gamma_2(y),\\ &u_x(1+0,y)=\beta_2(y)u_x(1-0,y)+\delta_2(y), \end{aligned} \tag{1.5}$$

bu yerda $\psi_i(y),\alpha_i,\beta_i,\gamma_i,\delta_i, i=1,2, \ \ \varphi(x)-$oldindan berilgan funksiyalar, $\psi_i'',\varphi',\alpha_i'',\beta_i'',\gamma_i'',\delta_i$ – uzluksiz.

II. ADABIYOTLAR TAHLILI

Parabola-giperbolik tipdagi tenglamalar uchun chegaraviy masalalarning o'zgarish chizig'i perpendikulyar bo'lgan xollari [1-3] adabiyotlarda, tipdagi o'zgarish chizig'i parallel bo'lgan xollari [4-5] adabiyotlarida o'rganilgan.

Ushbu masalaning boshqalardan farqi ([1-5]) parabolik sohada ikkinchi chegaraviy masala yechimidan foydalanib, $\nu_i(y)$, $(i=0,1)$ ga nisbatan $x=0$ va $x=1$ da Volterraning ikkinchi tur integral tenglamalari sistemasi olinadi va uning yechimining yagonaligi integral tenglamalar nazariyasidan kelib chiqadi.

Tip o‘zgarish chizig‘i bo‘yicha ishlarda ikkinchi turdagi Fredgolm integral tenglamalari sistemasi olingan va o‘ziga xosligi alohida isbotlangan.

III. NATIJALAR

2-§. Sohaning giperbolik qismida masalani yechish

Quyidagicha belgilashlar kiritamiz

$$\left.\begin{aligned} u(+0,y)&=\tau_0^+(y), \quad u_x(+0,y)=\nu_0^+(y),\\ u(-0,y)&=\tau_0^-(y), \quad u_x(-0,y)=\nu_0^-(y),\end{aligned}\right\}$$

$$\left.\begin{aligned} u(1+0,y)&=\tau_1^+(y), \quad u_x(1+0,y)=\nu_1^+(y),\\ u(1-0,y)&=\tau_1^-(y), \quad u_x(1-0,y)=\nu_1^-(y).\end{aligned}\right\}$$

Tenglamani yechishni D_2 sohadan boshlaymiz

$$u_{xx}-u_{yy}=0, \; u_{|x=-y}=\psi_1(y).$$

Dalamber formulasidan foydalanamiz

$$u(x,y)=\frac{\tau_0^-(y-x)+\tau_0^-(y+x)}{2}+\frac{1}{2}\int_{y-x}^{y+x}\nu_0^-(\eta)d\eta, \qquad (2.1)$$

D_2 sohada $x=-y$ ni qo‘yamiz

$$\tau_0^-(y)=f_1(y)+\int_0^y \nu_0^-(\eta)d\eta, \qquad (2.2)$$

tenglamaga ega bo‘lamiz. Bu yerda $f_1(y)=2\psi_1\left(\frac{y}{2}\right)-\psi_1(0)$ ga teng.

Dalamber formulasini D_3 soha uchun ishlatamiz

$$u(x,y)=\frac{\tau_1^+(x-y)+\tau_1^+(x+y)}{2}+\frac{1}{2}\int_{x-y}^{x+y}\nu_1^+(\eta)d\eta,$$

D_3 sohada $x=1+y$ ni qo‘yamiz

$$\tau_1^+(y)=f_2(y)+\int_y^1 \nu_1^+(\eta)d\eta, \qquad (2.3)$$

tenglamaga ega bo‘lamiz. Bu yerda $f_2(y)=2\psi_2\left(\frac{y-1}{2}\right)-\psi_2(0)$ ga teng.

3-§. Sohaning parabolik qismida masalani yechish

Parabolik sohada tenglamani yechamiz.

$$u_{xx}-u_y=0, \quad u(x,0)=\varphi(x), \quad u_x(0,y)=\nu_0^+(y), \quad u_x(1,y)=\nu_1^-(y). \quad (3.1)$$

(3.1) ko‘rinishdagi tenglamaning D_1 sohada yechimining ko‘rinishi quyidagiga teng [6-7].

$$u(x,y)=\int_0^1\varphi(\xi)G_2(x,y;\xi,0)d\xi-\int_0^y\nu_0(\eta)G_2(x,y;0,\eta)d\eta+\int_0^y\nu_1(\eta)G_2(x,y;1,\eta)d\eta \quad (3.2)$$

$$G_2(x,y;\xi,\eta)=\frac{1}{2\sqrt{\pi(y-\eta)}}\sum_{n=-\infty}^{+\infty}\left[\exp\left(-\frac{(x-\xi-2n)^2}{4(y-\eta)}\right)+\exp\left(-\frac{(x+\xi-2n)^2}{4(y-\eta)}\right)\right]$$

bu yerda $y>\eta$.

Ulashish shartlaridan foydalanamiz.

(3.2) formuladan foydalanib $u(+0,y)$ quyidagiga teng.

$$u(+0,y)=\int_0^1\varphi(\xi)G_2(0,y;\xi,0)d\xi-\int_0^y\nu_0^+(\eta)G_2(0,y;0,\eta)d\eta+\int_0^y\nu_1^-(\eta)G_2(0,y;1,\eta)d\eta,$$

$u(+0,y)=\tau_0^+(y)$ tenglikni inobatga olib

$$\tau_0^+(y)=-\int_0^y\frac{1}{\sqrt{\pi(y-\eta)}}\nu_0^+(\eta)d\eta-\int_0^y k_0(y,\eta)\nu_0^+(\eta)d\eta+\int_0^y K_1(y,\eta)\nu_1^-(\eta)d\eta+f_3(y)\quad(3.3)$$

Bu yerda

$$k_0(y,\eta)=\sum_{n=-\infty}^{+\infty}{}'\exp\left(-\frac{n^2}{y-\eta}\right),\ K_1(y,\eta)=G_2(0,y;1,\eta),$$

$$f_3(y)=\int_0^1\varphi(\xi)G_2(0,y;\xi,\eta)d\xi.$$

Keying oʻrinlarda $k(y,\eta)$ funksiya Grin funksiyasining $n\neq 0$ holatdagi. $K(y,\eta)$ funksiya esa Grin funksiyasining $n\in(-\infty,+\infty)$ holatdagiga teng.

(3.2) formuladan foydalanib $u(1-0,y)$ quyidagiga teng.

$$u(1-0,y)=\int_0^1\varphi(\xi)G_2(1,y;\xi,0)d\xi-\int_0^y\nu_0^+(\eta)G_2(1,y;0,\eta)d\eta+\int_0^y\nu_1^-(\eta)G_2(1,y;1,\eta)d\eta,$$

$u(1-0,y)=\tau_1^-(y)$ tenglikni inobatga olib

$$\tau_1^-(y)=\int_0^y\frac{1}{2\sqrt{\pi(y-\eta)}}\nu_1^-(\eta)d\eta+\int_0^y k_1(y,\eta)\nu_1^-(\eta)d\eta-\int_0^y K_0(y,\eta)\nu_0^+(\eta)d\eta+f_4(y)\quad(3.4)$$

Bu yerda

$$k_1(y,\eta)=\frac{1}{2\sqrt{\pi(y-\eta)}}\exp\left(-\frac{1}{y-\eta}\right)+\frac{1}{2\sqrt{\pi(y-\eta)}}\sum_{n=-\infty}^{+\infty}{}'\left[\exp\left(-\frac{n^2}{y-\eta}\right)+\exp\left(-\frac{(n-1)^2}{y-\eta}\right)\right]$$

$$K_0(y,\eta)=G_2(1,y;0,\eta),\quad f_4(y)=\int_0^1\varphi(\xi)G_2(1,y;\xi,0)d\xi.$$

4-§. Integral tenglamani hosil qilish

Quyidagicha tenglikni hisobga olib $\tau_0^-(y)=\alpha_1(y)\tau_0^+(y)+\gamma_1(y)$, tenglikka (2.2) va (3.3) tengliklarni mos ravishda oʻrniga qoʻyamiz.

$$f_1(y)+\int_0^y\nu_0^-(\eta)d\eta=-\alpha_1(y)\int_0^y\frac{1}{\sqrt{\pi(y-\eta)}}\nu_0^+(\eta)d\eta-\alpha_1(y)\int_0^y k_0(y,\eta)\nu_0^+(\eta)d\eta+$$

$$+\alpha_1(y)\int_0^y K_1(y,\eta)\nu_1^-(\eta)d\eta+\alpha_1(y)f_3(y)+\gamma_1(y). \qquad (4.1)$$

$\nu_0^-(y)=\beta_1(y)\nu_0^+(y)+\delta_1(y)$ tenglikni hisobga olib (4.1) tenglamaga qo‘yamiz.

$$f_1(y)+\int_0^y \beta_1(\eta)\nu_0^+(\eta)d\eta+\int_0^y \delta_1(\eta)d\eta=-\alpha_1(y)\int_0^y \frac{1}{\sqrt{\pi(y-\eta)}}\nu_0^+(\eta)d\eta-$$

$$-\alpha_1(y)\int_0^y k_0(y,\eta)\nu_0^+(\eta)d\eta+\alpha_1(y)\int_0^y K_1(y,\eta)\nu_1^-(\eta)d\eta+\alpha_1(y)f_3(y)+\gamma_1(y),$$

$$\int_0^y \frac{\nu_0^+(\eta)d\eta}{(y-\eta)^{\frac{1}{2}}}=-\frac{\sqrt{\pi}}{\alpha_1(y)}\int_0^y \beta_1(\eta)\nu_0^+(\eta)d\eta-\sqrt{\pi}\int_0^y k_0(y,\eta)\nu_0^+(\eta)d\eta+$$

$$+\sqrt{\pi}\int_0^y K_1(y,\eta)\nu_1^-(\eta)d\eta+\frac{\sqrt{\pi}}{\alpha_1(y)}f_5(y). \qquad (4.2)$$

Bu yerda $f_5(y)=\alpha_1(y)f_3(y)+\gamma_1(y)-f_1(y)-\int_0^y \delta_1(\eta)d\eta$

Quyidagicha Abel integralidan foydalanamiz.

$$\int_0^y \frac{\varphi(t)dt}{(y-t)^\alpha}=f(y),\quad 0<\alpha<1 \quad\Rightarrow\quad \varphi(y)=\frac{\sin\pi\alpha}{\pi}\frac{d}{dy}\int_0^y \frac{f(s)ds}{(y-s)^{1-\alpha}},$$

$$\alpha=\frac{1}{2},\quad\Rightarrow\quad \varphi(y)=\frac{1}{\pi}\frac{d}{dy}\int_0^y \frac{f(s)ds}{(y-s)^{\frac{1}{2}}}.$$

Abel integralini (4.2) tenglikka qo‘llaymiz.

$$\nu_0^+(y)=-\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{ds}{\alpha_1(s)(y-s)^{\frac{1}{2}}}\int_0^s \beta_1(\eta)\nu_0^+(\eta)d\eta-$$

$$-\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{ds}{(y-s)^{\frac{1}{2}}}\int_0^s k_0(s,\eta)\nu_0^+(\eta)d\eta+\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{ds}{(y-s)^{\frac{1}{2}}}\int_0^s K_1(s,\eta)\nu_1^-(\eta)d\eta+$$

$$+\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{f_5(s)}{\alpha_1(s)(y-s)^{\frac{1}{2}}}ds. \qquad (4.3)$$

Quyidagicha Dirixle formulasidan foydalanamiz

$$\int_0^y dt\int_0^t d\eta=\int_0^y d\eta\int_\eta^y dt.$$

Dirixle formulasini (4.3) tenglamaga qo‘llaymiz.

$$\nu_0^+(y)=-\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \beta_1(\eta)\nu_0^+(\eta)d\eta\int_\eta^y \frac{ds}{\alpha_1(s)(y-s)^{\frac{1}{2}}}-$$

$$-\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \nu_0^+(\eta)d\eta\int_\eta^y \frac{k_0(s,\eta)}{(y-s)^{\frac{1}{2}}}ds+\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \nu_1^-(\eta)d\eta\int_\eta^y \frac{K_1(s,\eta)}{(y-s)^{\frac{1}{2}}}ds+$$

$$+\frac{\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{f_5(s)}{\alpha_1(s)(y-s)^{\frac{1}{2}}}ds \qquad (4.4)$$

Ba'zi oddiy hisob-kitoblarni bajarib, biz ikkinchi turdagi Volterra integral tenglamasiga ega bo'lamiz

$$\nu_0^+(y)+\int_0^y K_2(y,\eta)\nu_0^+(\eta)d\eta+\int_0^y K_3(y,\eta)\nu_1^-(\eta)d\eta=F_1(y). \qquad (4.5)$$

Bu yerda

$$K_2(y,\eta)=\frac{\sqrt{\pi}}{2\pi}\frac{\beta_1(\eta)}{(y-\eta)^{\frac{1}{2}}}\int_0^1 \frac{dz}{\alpha_1(\eta+(y-\eta)z)(1-z)^{\frac{1}{2}}}-$$

$$-\frac{\sqrt{\pi}}{\pi}\beta_1(\eta)(y-\eta)^{\frac{1}{2}}\int_0^1 \frac{\alpha_{1y}{}'(\eta+(y-\eta)z)}{\alpha^2{}_1(\eta+(y-\eta)z)}\frac{z}{(1-z)^{\frac{1}{2}}}dz+$$

$$+\frac{\sqrt{\pi}}{2\pi}\frac{1}{(y-\eta)^{\frac{1}{2}}}\int_0^1 \frac{k_0(\eta+(y-\eta)z,\eta)}{(1-z)^{\frac{1}{2}}}dz+\frac{\sqrt{\pi}}{\pi}(y-\eta)^{\frac{1}{2}}\int_0^1 \frac{k_{oy}{}'(\eta+(y-\eta)z,\eta)z}{(1-z)^{\frac{1}{2}}}dz$$

,

$$K_3(y,\eta)=-\frac{\sqrt{\pi}}{2\pi}\frac{1}{(y-\eta)^{\frac{1}{2}}}\int_0^1 \frac{K_1(\eta+(y-\eta)z,\eta)dz}{(1-z)^{\frac{1}{2}}}-\frac{\sqrt{\pi}}{\pi}(y-\eta)^{\frac{1}{2}}\int_0^1 \frac{K_{1y}'(\eta+(y-\eta)z,\eta)zdz}{(1-z)^{\frac{1}{2}}}$$

$$F_1(y)=\frac{\sqrt{\pi}}{2\pi}\frac{1}{y^{\frac{1}{2}}}\int_0^1 \frac{f_5(yz)}{\alpha_1(yz)(1-z)^{\frac{1}{2}}}dz+\frac{\sqrt{\pi}}{\pi}y^{\frac{1}{2}}\int_0^1 \frac{f_{5y}'(yz)\alpha_1(yz)-f_5(yz)\alpha_{1y}'(yz)}{\alpha_1^2(yz)}\frac{z}{(1-z)^{\frac{1}{2}}}dz$$

Quyidagicha ulashish shartini hisobga olib $\tau_1^+(y)=\alpha_2(y)\tau_1^-(y)+\gamma_2(y)$, tenglikka (2.3) va (3.4) tengliklarni mos ravishda o'rniga qo'yamiz.

$$f_2(y)+\int_y^1 \nu_1^+(\eta)d\eta=\alpha_2(y)\int_0^y \frac{\nu_1^-(\eta)d\eta}{2\sqrt{\pi(y-\eta)}}+\alpha_2(y)\int_0^y k_1(y,\eta)\nu_1^-(\eta)d\eta-$$

$$-\alpha_2(y)\int_0^y K_0(y,\eta)\nu_0^+(\eta)d\eta+\alpha_2(y)f_4(y)+\gamma_2(y), \qquad (4.6)$$

$\nu_1^+(y)=\beta_2(y)\nu_1^-(y)+\delta_2(y)$ tenglikni hisobga olib (4.6) tenglamaga qo'yamiz.

$$f_2(y)+\int_y^1 \beta_2(\eta)\nu_1^-(\eta)d\eta+\int_y^1 \delta_2(\eta)d\eta=\alpha_2(y)\int_0^y \frac{\nu_1^-(\eta)d\eta}{2\sqrt{\pi(y-\eta)}}+$$

$$+\alpha_2(y)\int_0^y k_1(y,\eta)\nu_1^-(\eta)d\eta-\alpha_2(y)\int_0^y K_0(y,\eta)\nu_0^+(\eta)d\eta+\alpha_2(y)f_4(y)+\gamma_2(y),$$

$$\int_0^y \frac{\nu_1^-(\eta)d\eta}{(y-\eta)^{\frac{1}{2}}} = \frac{2\sqrt{\pi}}{\alpha_2(y)}\int_y^1 \beta_2(\eta)\nu_1^-(\eta)d\eta - 2\sqrt{\pi}\int_0^y k_1(y,\eta)\nu_1^-(\eta)d\eta +$$

$$+2\sqrt{\pi}\int_0^y K_0(y,\eta)\nu_0^+(\eta)d\eta + \frac{2\sqrt{\pi}}{\alpha_2(y)}\int_y^1 \delta_2(\eta)d\eta + \frac{2\sqrt{\pi}}{\alpha_2(y)} f_2(y) -$$

$$-\frac{2\sqrt{\pi}}{\alpha_2(y)}\alpha_2(y) f_4(y) - \frac{2\sqrt{\pi}}{\alpha_2(y)}\gamma_2(y), \qquad (4.7)$$

quiydagicha belgilash kiritamiz.

$$f_6(y) = \frac{2\sqrt{\pi}}{\alpha_2(y)}\left[\int_y^1 \delta_2(\eta)d\eta + f_2(y) - \alpha_2(y) f_4(y) - \gamma_2(y)\right].$$

Abel integralini (4.7) tenglik uchun ishlatamiz

$$\nu_1^-(y) = \frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{ds}{\alpha_2(s)(y-s)^{\frac{1}{2}}}\int_s^1 \beta_2(\eta)\nu_1^-(\eta)d\eta -$$

$$-\frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{ds}{(y-s)^{\frac{1}{2}}}\int_0^s k_1(s,\eta)\nu_1^-(\eta)d\eta + \frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{ds}{(y-s)^{\frac{1}{2}}}\int_0^s K_0(s,\eta)\nu_0^+(\eta)d\eta +$$

$$+\frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \frac{f_6(s)ds}{(y-s)^{\frac{1}{2}}}. \qquad (4.8)$$

Dirixle formulasini (4.8) tenglamaga qo‘llaymiz.

$$\nu_1^-(y) = \frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_y^1 \beta_2(\eta)\nu_1^-(\eta)d\eta\int_0^y \frac{ds}{\alpha_2(s)(y-s)^{\frac{1}{2}}} -$$

$$-\frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \nu_1^-(\eta)d\eta\int_\eta^y \frac{k_1(s,\eta)}{(y-s)^{\frac{1}{2}}}ds + \frac{2\sqrt{\pi}}{\pi}\frac{d}{dy}\int_0^y \nu_0^+(\eta)d\eta\int_\eta^y \frac{K_0(s,\eta)}{(y-s)^{\frac{1}{2}}}ds +$$

$$+\frac{1}{\pi}\frac{d}{dy}\int_0^y \frac{f_6(s)ds}{(y-s)^{\frac{1}{2}}}. \qquad (4.9)$$

IV. MUHOKAMA

$$\nu_1^-(y) + \int_y^1 K_4(y,\eta)\nu_1^-(\eta)d\eta + \int_0^y K_5(y,\eta)\nu_1^-(\eta)d\eta + \int_0^y K_6(y,\eta)\nu_0^+(\eta)d\eta = F_2(y) \qquad (4.10)$$

Bu yerda

$$K_4(y,\eta) = -\frac{\sqrt{\pi}}{\pi}\beta_2(\eta)\frac{1}{y^{\frac{1}{2}}}\int_0^1 \frac{dz}{\alpha_2(yz)(1-z)^{\frac{1}{2}}} - \frac{2\sqrt{\pi}}{\pi}\beta_2(\eta) y^{\frac{1}{2}}\int_0^1 \frac{\alpha_{2_y}{}'(yz)zdz}{\alpha_2^2(yz)(1-z)^{\frac{1}{2}}},$$

$$K_5(y,\eta)=\frac{\sqrt{\pi}}{\pi}\frac{1}{(y-\eta)^{\frac{1}{2}}}\int_0^1\frac{k_1(\eta+(y-\eta)z,\eta)dz}{(1-z)^{\frac{1}{2}}}+\frac{2\sqrt{\pi}}{\pi}(y-\eta)^{\frac{1}{2}}\int_0^1\frac{k'_{1y}(\eta+(y-\eta)z,\eta)zdz}{(1-z)^{\frac{1}{2}}}$$

$$K_6(y,\eta)=-\frac{\sqrt{\pi}}{\pi}\frac{1}{(y-\eta)^{\frac{1}{2}}}\int_0^1\frac{K_0(\eta+(y-\eta)z,\eta)dz}{(1-z)^{\frac{1}{2}}}-\frac{2\sqrt{\pi}}{\pi}(y-\eta)^{\frac{1}{2}}\int_0^1\frac{K'_{0y}(\eta+(y-\eta)z,\eta)zdz}{(1-z)^{\frac{1}{2}}},$$

$$F_2(y)=\frac{1}{2\pi}\frac{1}{y^{\frac{1}{2}}}\int_0^1\frac{f_6(yz)dz}{(1-z)^{\frac{1}{2}}}+\frac{1}{\pi}y^{\frac{1}{2}}\int_0^1\frac{f'_{6y}(yz)zdz}{(1-z)^{\frac{1}{2}}}.$$

(4.10) tenglikni quyidagi ko‘rinishda yozib olamiz.

$$\nu_1^-(y)+\int_y^1 K_7(y,\eta)\nu_1^-(\eta)d\eta+\int_0^y K_6(y,\eta)\nu_0^+(\eta)d\eta=F_2(y). \qquad (4.11)$$

$$K_7(y,\eta)=K_4(y,\eta)+\begin{cases}K_5(y,\eta);0<\eta<y,\\ 0; \qquad y<\eta<1.\end{cases}$$

V. XULOSALAR

Qo‘yilgan masalaning bir qiymatli yechimiga ekvivalent bo‘lgan Volterraning ikkinchi tur integral tenglamalar sistemasining yechimi (4.5) va (4.11) keladi, bundan esa bir qiymatli yechimini hosil qiladi.

(4.5) va (4.11) Volterra tipidagi ikkinchi tur integral tenglamalarda o‘ng tomondagi funksiyalar va tenglamalarning yadrosi $\alpha=\frac{1}{2}$ darajadagi maxsuslikka ega. Shuning uchun bu tenglamalar sistemasi bir qiymatli yechimga ega. Bunda esa qo‘yilgan masalani bir qiymatli yechimga ega ekanligi kelib chiqadi.

VI. ADABIYOTLAR RO‘YXATI

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