UCHINCHI TARTIBLI KARRALI XARAKTERISTIKALI TENGLAMA UCHUN BIR CHIZIQSIZ CHEGARAVIY MASALA HAQIDA

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Annotasiya. *Ushbu maqolada egri chiziqli sohada karrali xarakteristikaga ega uchinchi tartibli chiziqli bo‘lmagan tenglama uchun bir chiziqli bo‘lmagan chegaraviy masala o‘rganilgan. Masala yechimining yagonaligi va mavjudligi isbotlangan. Masala yechimining yagonaligi energiya integrali usulidaga ba’zi elementar tengsizliklarni tadbiq qilish orqali isbotlangan. Masala yechimining mavjudligini isbotlasshda yordamchi masala qaralgan va bu nasala uchun Grin funksiyasi qurilgan. Yordamchi masala yechimi yordamida berilgan masala Hammershteyn tipidagi chiziqsiz integral tenglamalar tizimiga keltirilgan va u ketma-ket yaqinlashish usuli yordamida yechilgan.*

Kalit so‘zlar: *chiziqsizlik, yagonalik, mavjudlik, chiziqsiz integral tenglamalar tizimi, ketma-ket yaqinlashish usuli.*

I. KIRISH

Karrali xarakteristikali (MC) deb ataluvchi quyidagi tenglama

$$L(u) \equiv u_{xxx} - u_y = f(x, y, u, u_x, u_{xx}), \quad (MC)$$

kam o‘rganilgan toq tartibli tenglamalar sinfiga kiradi [1].

Karrali xarakteristikali (MC) tenglamalarning xususiy hollari fizika, mexanika va meditsinaning turli masalalarida ko‘p uchraydi. Shuning uchun ular katta nazariy va amaliy ahamiyatga ega.

Bu tenglama Korteweg-de Vris tenglamasi (KdV) deb atluvchi quyidagi

$$u_y + uu_x + \beta u_{xxx} = 0 \qquad (KdV)$$

mashhur tenglamani o‘z ichiga qamrab olgan bo‘lib, u ko‘plab tadqiqotchilarning tadqiqot ob’ektidir. Bu tenglama kuchsiz dispersiyali muhitda chiziqsiz to‘lqin tarqalishini o‘rganishda muhim o‘rin tutadi[2 - 4].

Korteweg-de Vris (KdV) tenglamasi yuqori chastotali dispersiyaga ega bo‘lgan muhitda kuchsiz chiziqsiz uzun to‘lqinli harakatlarning evolyutsiyasini tavsiflaydi. KdV tenglamasi suyuqliklar dinamikasida (sayoz suvda gravitatsion (tortishivchan) to‘lqinlarini va chiziqli bo‘lmagan Rossbi to‘lqinlarini modellashtirishda), plazma fizikasida (ion-akustik to‘lqinlarni tavsiflashda), elektrotexnikada (chiziqsiz zanjirlar tahlilida) va hatto epidemiologiyada(epidemiya davrida kasallangan shaxslar sonining evolyutsiyasini modellashtirishda) va boshqalar kabi turli sohalarda tadbiqlarinini topdi [2 – 5].

II. ADABIYOTLAT TAHLILI

So‘nggi yillarda turli ilmiy sohalardagi tadqiqotchilar Korteweg-de Vries tenglamasini tobora ko‘proq o‘rganmoqdalar [6 - 14].

Korteweg-de Vris (KdV) tenglamasi uchun chekli sohadagi boshlang'ich-chegaraviy masalaga bag'ishlangan birinchi maqola Bubnov B. A tomonidan 1979 yilda [6] ishda o‘rganilgan bo‘lib, u umumiy chegaraviy shartlar asosida boshlang'ich-chegaraviy masalasini ko‘rib chiqqan. Shu vaqtdan boshlab ko‘plab mualliflar mavjud natijalarni yaxshilash va yangi natijalarga erishish ustida ishlamoqdalar [7 - 14].

Ushbu va boshqa amaliy tadbiqlarni hisobga olgan holda, karali xarakterli uchinchi tartibli tenglamalar uchun chegaraviy masalalarni o‘rganish maqsadga muvofiqdir. Uchinchi tartibli karali xarakterli chiziqli tenglama uchun ba'zi chiziqli chegaraviy masalalar [1, 10 - 11] ishlarda ko‘rib chiqilganlan. [10] tadqiqotlarda karrali xarakterli uchinchi tartibli chizsiz tenglama uchun chiziqli chegaraviy masala o‘rganilgan. Karali xarakterli uchinchi tartibli chiziqli tenglama uchun chiziqli bo‘lmagan chegaraviy masala [12] maqolada tahlil qilingan. [13 - 14] ishlarda egri chiziqli sohada karrali xarakteristikali chiziqli bo‘lmagan uchinchi tartibli tenglama uchun chiziqli bo‘lmagan chegaraviy masalalar tadqiq etilgan. Ushbu maqolada egri chiziqli sohada karrali xarakteristikaga ega uchinchi tartibli chiziqli bo‘lmagan tenglama uchun bir chiziqli bo‘lmagan chegaraviy masala o‘rganilgan.

III. NATIJALAR

Masalaning qo‘yilishi

A masala. $D=\{(x,y): h_1(y)<x<h_2(y), 0<y\le 1\}$ sohada shunday u(x,y) funnksiya topilsinki, u quyidagi xossalarga ega bo‘lsin:

1) $u(x,y)\in C^{3,1}_{x,y}(D)\cap C^{2,0}_{x,y}(\overline{D})$;

2) D sohada

$$L(u)\equiv u_{xxx}-u_y=f(x,y,u(x,y),u_x(x,y)) \quad (1)$$

tenglamaning regulyar yechimi bo‘lsin;

3) quyidagi shartlarni qanoatlantirdin

$$u(x,0)=u_0(x),\quad h_1(0)\le x\le h_2(0), \quad (2)$$

$$u(h_1(y),y)=\varphi(y), 0\le y\le 1, \quad (3)$$

$$u_x(h_1(y),y)=\psi(y), 0\le y\le 1, \quad (4)$$

$$u_{xx}(h_2(y),y)+u_x(h_2(y),y)=g(u(h_2(y),y),y), 0\le y\le 1, \quad (5)$$

va quyidagi kelishuvlik shartlari o‘rinli bo‘lsin

$$u_0''(h_2(0))+u_0'(h_2(0))=g(u_0(h_2(0)),0)\ ,\quad u_0(h_1(0))=\varphi(0)\ ,\quad u_0'(h_2(0))=\psi(0)\ .$$

Yechimning yagonaligi

Teorema (Yechimning yagonaligi). Agar $h_r(y)\in C^1[0;1], r=1,2$ va argumentning $0\le y\le 1$ qiymatlarida har qanday $|u|<\infty$ uchun $g(u,y)$, $f(x,y,u,u_x)$ funksiyalar o‘z argumentlarining uzluksiz funksiyalar bo‘lib, quyidagi

$$|g(\tau_1,y)-g(\tau_2,y)|\le l|\tau_1-\tau_2|, \quad (6)$$

$$|f(x,y,u_1,u_{1x})-f(x,y,u_2,u_{2x})|\le L\{|u_1-u_2|+|u_{1x}-u_{2x}|\}, \quad (7)$$

$$2l+L+1+h_2'(y)\le 0, \quad (8)$$

$$0 < L \le \frac{N}{3}, \qquad (9)$$

shartlarni qanoatlantirsin, bunda $\tau(y) = u(h_2(y), y)$ va $l > 0$, $L > 0$, $N > 0$ o'garmas sonlar. U holda A masalaning yechimi yagonadir .

Isbot. Masalani yechishning yagonaligi energiya integrali usulida, ba'zi elementar tengsizliklardan foydalangan holda isbotlanadi.

Ko'rib chiqilayotgan masalaning ikkita turli u_1 va u_2 yechilari mavjud bo'lsin deb faraz qilaylik. Bu yechimlarning farqini qaraylik $w = u_1 - u_2$. w funksiyaga nisbatan quyidagi masalaga ega bo'lamiz:

$$L(w) \equiv w_{xxx} - w_y = f(x, y, u_1, u_{1x}) - f(x, y, u_2, u_{2x}) \qquad (1_0)$$

$$w(x, 0) = 0, \; h_1(0) \le x \le h_2(0), \qquad (2_0)$$

$$w(h_1(y), y) = 0, \; 0 \le y \le 1 \qquad (3_0)$$

$$w_x(h_1(y), y) = 0, \; 0 \le y \le 1 \qquad (4_0)$$

$$w_{xx}(h_2(y), y) + \alpha w_x(h_2(y), y) = g(u_1(h_2(y), y), y) - g(u_2(h_2(y), y), y), \; 0 \le y \le 1 \qquad (5_0)$$

$w(x, y) \equiv 0$ ekanligini ko'rsatamiz.

Quyidagi

$$vwL(w) \equiv vw(w_{xxx} - w_y) = vw(f(x, y, u_1, u_{1x}) - f(x, y, u_2, u_{2x}) \qquad (10)$$

ayniyatni D soha bo'yicha integrallab, bunda $v = \exp(-x - (N+1)y)$, (2_0) - (5_0) shartlarni inobatga olib , quyidagiga ega bo'lamiz:

$$\int_0^1 (g(u_1) - g(u_2)) vw\Big|^{x=h_2(y)} dy - \frac{1}{2}\int_0^1 vw_x^2\Big|^{x=h_2(y)} dy + \frac{1}{2}\int_0^1 (h_2'(y) + 1) vw^2\Big|^{x=h_2(y)} dy - \frac{3}{2}\iint_D v\, w_x^2 dxdy +$$
$$+ \frac{1}{2}\iint_D (v_y - v_{xxx}) w^2 dxdy - \frac{1}{2}\int_{h_1(y)}^{h_2(y)} vw^2\Big|^{y=1} dx = \iint_D (f(x, y, u_1, u_{1x}) - f(x, y, u_2, u_{2x})) wvdxdy. \qquad (11)$$

Quyidagi belgilashni kiritamiz:

$$I = \frac{1}{2}\int_0^1 vw_x^2\Big|^{x=h_2(y)} + \frac{1}{2}\int_{h_1(y)}^{h_2(y)} vw^2\Big|^{y=1} dx + \frac{3}{2}\iint_D vw_x^2 dxdy \ge 0. \qquad (12)$$

(12) belgilashga ko'ra (11) munosabatdan quyidagi tenglikni hosil qilamiz

$$I = \int_0^1 (g(u_1) - g(u_2)) vw\Big|^{x=h_2(y)} dy + \frac{1}{2}\int_0^1 (h_2'(y) + 1) vw^2\Big|^{x=h_2(y)} dy +$$
$$+ \frac{1}{2}\iint_D (v_y - v_{xxx}) w^2 dxdy - \iint_D (f(x, y, u_1, u_{1x}) - f(x, y, u_2, u_{2x})) wvdxdy.$$

(6) va (7) shartlar asosida esa quyidagi munosabatga ega bo'lamiz:

$$I \le \frac{1}{2}\int_0^1 (2l + L + h_2'(y) + 1) w^2 v\Big|_{x=h_1(y)} dy + \frac{1}{2}\iint_D (3L - N) w^2 vdxdy .$$

(8) va (9) shartlar bajarilganda esa, oxirgi munosabatdan quyidagi tengsizlikka ega bo'lamiz $I \le 0$. Demak, $I = 0$.

Shuningdek, (12) munosabatdan quyidagi shartlarga ega bo'lamiz:

$$w_x(h_2(y), y) = 0, \; 0 \le y \le 1, \; w(x,1) = 0, h_1(1) \le x \le h_2(1), \; w_x(x, y) = 0, \; (x, y) \in D.$$

Demak, $\forall (x, y) \in D$ $uchun$ $w(x, y) = p(y)$.

$w(h_2(y),y)=0$, $0\le y\le 1$, bo'lgani uchun $p(y)\equiv 0.$

$w(x,y)$ funksiyaning $\overline{D}$ sohada uzluksizligidan esa $w(x,y)=0$ munosabayni hosil qilamiz.

Yechimning mavjudligi

A masala yechimi mavjudligini isbotlashdan oldin quyidagi yordamchi masalani o'rganamiz.

B masala. D sohada

$$L(u)\equiv u_{xxx}-u_y=f(x,y) \qquad (1_1)$$

tenglamaning shunday $u(x,y)\in C^{3,1}_{x,y}(D)\cap C^{2,0}_{x,y}(\overline{D})$

regulyar yechimi topilsinki, u quyidagi shartlarni qanoatlantirsin

$$u(x,0)=u_0(x),\quad h_1(0)\le x\le h_2(0), \qquad (2)$$

$$u(h_1(y),y)=\varphi(y), 0\le y\le 1, \qquad (3)$$

$$u_x(h_1(y),y)=\psi(y), 0\le y\le 1, \qquad (4)$$

$$u_{xx}(h_2(y),y)=\psi_1(y), 0\le y\le 1, \qquad (5_1)$$

B masala uchun Grin funksiyasi [20] ishda qurilgan va u quyidagi shaklga ega:

$$u(x,y)=\frac{1}{\pi}\int_0^y Gu_{\xi\xi}\Big|_{\xi=h_2(\eta)}d\eta+\frac{1}{\pi}\int_0^y G_\xi u_\xi\Big|_{\xi=h_1(\eta)}d\eta+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)} Gu\Big|_{\eta=0}d\xi+$$
$$-\frac{1}{\pi}\int_0^y\left(G_{\xi\xi}+h'_1(\eta)G\right)u\Big|_{\xi=h_1(\eta)}d\eta-\frac{1}{\pi}\iint_D G(x,y;\xi,\eta)f(\xi,\eta)d\xi d\eta, \qquad (13)$$

bu yerda $G(x,y;\xi,\eta)=U(x,y;\xi,\eta)-W(x,y;\xi,\eta)$.

(13) formula B masala yechimini ifodalaydi.

$G(x,y;\xi,\eta)$ funksiya B masalaning Grin funksiyasi deb ataladi.

$G(x,y;\xi,\eta)$ funksiya uchun $U(x,y;\xi,\eta)$ funksiya baholari o'rinlidir([1]).

$$\left.\begin{array}{l} U(x.y;\xi,\eta)=\begin{cases}(y-\eta)^{-\frac{1}{3}}f\left(\dfrac{x-\xi}{(y-\eta)^{\frac{1}{3}}}\right), & x\ne\xi,\ y>\eta,\\ 0, & y\le\eta.\end{cases}\\ V(x.y;\xi,\eta)=\begin{cases}(y-\eta)^{-\frac{1}{3}}\varphi\left(\dfrac{x-\xi}{(y-\eta)^{\frac{1}{3}}}\right), & x>\xi,\ y>\eta,\\ 0, & y\le\eta.\end{cases}\end{array}\right\} (14)$$

Bu yerda

$$f(t)=\int_0^{+\infty}\cos(\lambda^3-\lambda t)d\lambda,\ -\infty<t<+\infty,\ \varphi(t)=\int_0^\infty\left[\exp\left(-\lambda^3-\lambda t\right)+\sin\left(\lambda^3-\lambda t\right)\right]d\lambda,\ \ t=\frac{x-\xi}{(y-\eta)^{1/3}}.$$

$f(t),\ \varphi(t)$ Airy funksiyalari deb atalib, quyidayi tenglamani qanoatlantiradi ([1])

$$z''(t)+\frac{1}{3}tz(t)=0.$$

Airy funksiyalari uchun qyidagi munosabatlar o'rinlidir

$$\int_0^{+\infty} f(t)dt = \frac{2}{3}\pi,\ \int_0^{+\infty} \varphi(t)dt = 0,\ \int_{-\infty}^{0} f(t)dt = \frac{\pi}{3}. \qquad (15)$$

$U(x,y;\xi,\eta)$, $V(x,y;\xi,\eta)$ funksiyalar uchun quyidagi baholar o'rinlidir:

agar $\dfrac{x-\xi}{(y-\eta)^{1/3}} \to +\infty,\ i+j\geq 1,\ C>0,\ C_1>0,$

$$|U(x,y;\xi,\eta)| < \frac{C}{(y-\eta)^{1/3}},\quad \left.\begin{array}{l}\left|\dfrac{\partial^{i+j}U(x,y;\xi,\eta)}{\partial x^i \partial y^j}\right| \\ \left|\dfrac{\partial^{i+j}V(x,y;\xi,\eta)}{\partial x^i \partial y^j}\right|\end{array}\right\} < C_1 \frac{|x-\xi|^{\frac{2i+6j-1}{4}}}{|y-\eta|^{\frac{2i+6j-1}{4}}}, \qquad (16)$$

agar $\dfrac{x-\xi}{(y-\eta)^{1/3}} \to -\infty,\ i+j\geq 1,\ C_2>0,\ C_3>0$

$$\left|\frac{\partial^{i+j}U(x,y;\xi,\eta)}{\partial x^i \partial y^j}\right| < \frac{C_2}{|y-\eta|^{\frac{i+3j+1}{4}}} \exp\left(-C_3 \frac{|x-\xi|^{\frac{3}{2}}}{|y-\eta|^{\frac{1}{2}}}\right), \qquad (17)$$

Endi A maslaga o'rganamiz.

Teorema. Yagonalik teoremasining shartlari bilan bir qatorda quyidagi shartlar ham bajarilsin:

$$u_0(x) \in C^3[h_1(0;h_2(0))],\ \varphi(y) \in C^1[0,1],\quad \psi(y) \in C[0,1],$$

$$\frac{\partial}{\partial y}(f(x,y,u,u_x)) \in C(\overline{D}),\ f(x,0,u(x,0),u_x(x,0)) = 0,$$

va $\forall y \in [0;1]$ va $\forall |\tau(y)| < \infty$ chun quyidagi tengsizliklar bajarilsin:

$$|g(\tau(y),y)| < N,\ |g_y(\tau(y),y)| < N_1,\ |g_\tau(\tau(y),y)| < N_2,$$

$(x,y) \in D$ uchun $f(x,y,u) \in C(D)$ va $\forall |u| < \infty$ uchun

$$|f(x,y,u,u_x)| < M,\ |f_x(x,y,u,u_x)| < M_1,\ |f_y(x,y,u,u_x)| < M_2,$$

$$|f_u(x,y,u,u_x)| < M_3,\ |f_{u_x}(x,y,u,u_x)| < M_4.$$

U holda (1) - (5) masala yechimi mavjuddir.

Isbot. (13) munosabatdan quyidagiga ega bo'lamiz

$$u(x,y) = \frac{1}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta - \frac{1}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)u(h_2(\eta),\eta)d\eta -$$
$$-\frac{1}{\pi}\iint_D G(x,y;\xi,\eta)f(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta))d\xi d\eta + H(x,y), \qquad (18)$$

bu yerda

$$H(x,y) = \frac{1}{\pi}\int_0^y \left(G_{\xi\xi}(x,y;h_1(\eta),\eta) + h_1'(\eta)G(x,y;h_1(\eta),\eta)\right)\varphi(\eta)d\eta + \frac{1}{\pi}\int_0^y G_\xi(x,y;h_1(\eta),\eta)\psi(\eta)d\eta +$$
$$+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)} G(x,y;\xi,0)u_0(\xi)d\xi.$$

Quyidagi belgilashni kiritmiz: $u(h_2(y),y)=\nu(y)$. (18) munosabatda $x\to h_2(y)$ limitga o'tib quyidagi tenglikka ega bo'lamiz

$$\tau(y)=\frac{1}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta-\frac{1}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)\nu(\eta)d\eta-$$
$$-\frac{1}{\pi}\iint_D G(h_2(y),y;\xi,\eta)f(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta))d\xi d\eta+H(h_2(y),y). \quad (19)$$

Shuningdek, (18) munosabatni x bo'yicha bir marta differensiallab va $x\to h_2(y)$ limitga o'tib, quyidagi munosabatni hosil qilamiz:

$$\nu(y)=\frac{1}{\pi}\int_0^y G_x(h_2(y),y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta-\frac{1}{\pi}\int_0^y G_x(h_2(y),y;h_2(\eta),\eta)\nu(\eta)d\eta-$$
$$-\frac{1}{\pi}\iint_D G_x(h_2(y),y;\xi,\eta)f(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta))d\xi d\eta+H_x(h_2(y),y) \quad (20)$$

(18) - (20) sistema $\tau(y),\ \nu(y),\ u(x,y)$ noma'lumlarga nisbatan Hammerstein tipidagi chiziqsiz integral tenglamalar sistemasidir .

Ketma-ket yaqinlashish usuli yordamida ushbu sistemaning yagona yechuvchanligini isbotlaymiz.

$g(\tau(y),y)$ va $f(x,y,u(x,y))$ ma'lum funksiyalar bo'lsin, u holda (20) munosabatdan $\nu(y)$ noma'lumni topish mumkin.

$$\nu(y)=-\frac{1}{\pi}\int_0^y G_x(h_2(y),y;h_2(\eta),\eta)\nu(\eta)d\eta+F_2(y), \quad (21)$$

$$F_2(y)=\frac{1}{\pi}\int_0^y G_x(h_2(y),y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta\ \ \frac{1}{\pi}\iint_D G_x(h_2(y),y;\xi,\eta)f(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta))d\xi d\eta+F_1(y).$$

$F_2(y)$ uzluksiz funksiyalar uchun (22) tenglamaning uzluksiz funksiyalar sinfida quyidagi ko'rinishdagi yagona yechim mavjudligini ko'rsatish mumkin

$$\nu(y)=-\frac{1}{\pi}\int_0^y R(y,\eta;\lambda)F_2(\eta)d\eta+F_2(y), \quad (22)$$

bu *yerda* $R(y,\eta;\lambda)$ yadroning rezolventasi.

Endi (19) munosabatda $f(x,y,u,u_x)$ ma'lum deb, $\tau(y)$ ni aniqlaymiz.

$$\tau(y)=\frac{1}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta+F_3(y), \quad (23)$$

$$F_3(y)=-\frac{1}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)F_2(\eta)d\eta+\frac{1}{\pi^2}\int_0^y\int_0^\eta G(h_2(y),y;h_2(\eta),\eta)R(\eta,s;\lambda)F_2(s)dsd\eta-$$
$$-\frac{1}{\pi}\iint_D G(h_2(y),y;\xi,\eta)f(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta))d\xi d\eta+F(y).$$

Bu $\tau(y)$ noma'lumga nisbatan Hammersten tipidagi chiziqsiz integral tenglamadir. (23) tenglamani ketma-ket yaqinlashish usuli bilan yechish mumkin.

Shunday K va N musbat sonlar mavjud bo'lsinki, ular uchun quyidagi munosabarlar o'rinli bo'lsin:

$$|F_3(y)| \le K, \; |g(\tau(y), y)| \le N, \; |\tau(y)| \le K+1. \tag{24}$$

Quyidagi rekurent munosabatni qaraylik

$$\left.\begin{aligned} &\tau^{(0)}(y) = F_3(y) \le |F_3(y)| \le K < K+1, \\ &\tau^{(n)}(y) = F_3(y) + \frac{1}{\pi}\int_0^y G(h_2(y), y; h_2(\eta), \eta) g(\tau^{(n-1)}(\eta), \eta) d\eta. \end{aligned}\right\} \tag{25}$$

tenglikda n = 1 deb (16) baholardan foydalanib, quyidagiga ega bo'lamiz

$$|\tau^{(1)}(y)| = K + CN\int_0^y (y-\eta)^{-1/3} d\eta = K + \frac{3}{2}CNy^{2/3}.$$

g(τ(y),y) funksiya τ(y) argumenti y϶[0,1] qiymatlar uchun (24) tengzisliklarni qanoatlantirishi uchun, bunda g chegaralgan, quyidagi shartning bajarish zarur

$$K + \frac{3}{2}CNy^{2/3} < K+1, \quad \text{yoki} \quad y < \left(\frac{2}{3CN}\right)^{3/2}. \tag{26}$$

n = 2 da (26) dan

$$|\tau^{(2)}(y)| \le K + \frac{3}{2}CNy^{2/3}.$$

munosabatga ko'ra esa

$$|\tau^{(2)}(y)| \le K+1. \tag{27}$$

ya'ni τ(y) argumentining takroriy (ikkinchi) yaqinlashishida g(τ(y),y) funksiya (24) chegaralangan sohadan chiqb ketmaydi.

To'liq induksiya usulini qo'llagan holda, agar (26) shart bajarilsa, ketma-ket yaqinlashishlarning hech biri hadi (24) sohani tark etmaydi degan xulosaga kelamiz.

Endi $\{\tau^{(n)}(y)\}$ ketma-ketlik limiti mavjudligini ko'rsatamiz.

Buning uchun quyidagi qatorning yaqinlashuvini isbotlash kifoya

$$\tau^{(0)}(y) + (\tau^{(1)}(y) - \tau^{(0)}(y)) + (\tau^{(2)}(y) - \tau^{(1)}(y)) + \ldots + (\tau^{(k)}(y) - \tau^{(k-1)}(y)) + \ldots \tag{27}$$

Qator hadlarning absolyut qiymatlarini baholaylik.

$$|\tau^{(k)} - \tau^{(k-1)}| \le \frac{C^k l^{k-1} N}{\pi^k} \prod_{q=1}^{k} B\left(\frac{2q+1}{3}, \frac{2}{3}\right) y^{k\frac{2}{3}}$$

bu yerda B(a,b) - beta funksiya, П - ko'paytma belgisi.

Ko'rinib turibdiki, (27) qator har bir hadining absolyut qiymati darajali qator mos hadlaridan katta emas:

$$\sum_{k=1}^{\infty} \frac{C^k l^{k-1} N}{\pi^k} \prod_{q=1}^{k} B\left(\frac{2q+1}{3}; \frac{2}{3}\right) y^{\frac{2k}{3}}. \tag{28}$$

Bu qatorni yaqinlashishini o'rnatamiz. D'Alembert mezonini qo'llab,

$$\lim_{k\to\infty}\left|\frac{a_k}{a_{k-1}}\right|=\lim_{k\to\infty}\frac{Cl}{\pi}B\left(\frac{2k+1}{3},\frac{2}{3}\right)=0.$$

(28) qator tekis yaqinlashgani uchun, integral belgisi ostida limitga o'tish mumkin

$$\tau(y)=\frac{1}{\pi}\int_0^y G(h_1(y),y;h_1(\eta),\eta)g(\tau(\eta),\eta)d\eta\ +E(y).$$

Biz yechimni $D_0=\{h_1(y)\le x\le h_2(y),\ 0<y\le y_0\}$ sohada mavjudligini ko'rsatdik, garchi masala $D=\{(x,y):h_1(y)<x<h_2(y),0<y\le 1\}$ sohada qoyilgan bo'lsa ham. D0 va D sohalar turlichadir.

Agar $y_0\ge 1$ bo'lsa, u holda masala to'liq yechilgandir. Agar y0 <1 bo'lsa, u holda yechimni davom ettirish mumkin. Bunga quyidagicha erishish mumkin. Masalani D1 =D\D0 sohada o'rganamiz..

Berilgan sohadagi masalani yechish uchun yuqorida aytib o'tilgan sxemadan foydalanib, chiziqli bo'lmagan Volterra integral tenglamasini hosil qilamiz.

$$\tau(y)=\frac{1}{\pi}\int_{y_0}^y G(h_1(y),y;h_1(\eta),\eta)g(\tau(\eta),\eta)d\eta\ +E(y).$$

Yuqoridagi kabi mulohazalarda bu integral tenglamani berilgan sohada ketma-ket yaqinlashish usulida yechish mumkin.

$$D_1=\{h_1(y)\le x\le h_2(y),\ y_0<y\le y_1\},$$

bu yerda

$$y_1\le\left(\frac{2}{3CM}\right)^{\frac{3}{2}}+y_0 \quad \text{yoki} \quad y_1\le 2\left(\frac{2}{3CM}\right)^{\frac{3}{2}}.$$

Agar shundan so'ng ham y < 1 bo'lsa, yuqoridagi jarayonni takrorlash mumkin. Har bir qadamda $\delta=y_{k+1}-y_k=\left(\frac{2}{3CM}\right)^{\frac{3}{2}}$ musbat qadamga yurganimiz uchun va bu qadam oldingisidan kichik emasligidan, chekli sondagi qadamdan so'ng D sohani to'liq qoplash mumkin.

Endi (19) tenglamani o'rganamiz.

$$u(x,y)=P(x,y)-\frac{1}{\pi}\iint_D G(x,y;\xi,\eta)f(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta))d\xi d\eta\,, \tag{29}$$

bu yerda

$$P(x,y)=-\frac{1}{\pi}\int_0^y G(x,y,h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta\ -\frac{1}{\pi}\int_0^y G(x,y,h_2(\eta),\eta)\nu(\eta)d\eta\ +\frac{1}{\pi}\int_0^y\left(G_{\xi\xi}(x,y,h_1(\eta),\eta)+h_1'(\eta)G(x,y,h_1(\eta),\eta)\right)\phi(\eta)d\eta+$$
$$+\frac{1}{\pi}\int_0^y G_\xi(x,y,h_1(\eta),\eta)\psi(\eta)d\eta+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)}G(x,y;\xi,0)u_0(\xi)d\xi.$$

(29) tenglamani ham ketma - ket yaqinlashish usulida yechamiz.

Quyidagi tengsizliklar o'rinli bo'lsin

$$|P(x,y)| \le N, \ |P_x(x,y)| \le N, \ |f(x,y,u,u_x)| \le M, \ |u| \le N+1, \ |u_x| \le N+1. \quad (30)$$

Quyidagi rekurent munosabatlarni qaraylik

$$\left.\begin{aligned} &u^{(0)}(x,y) = P(x,y), \ u_x^{(0)}(x,y) = P_x(x,y), \\ &u^{(k)}(x,y) = P(x,y) - \frac{1}{\pi}\iint_D G(x,y;\xi,\eta) f\left(\xi,\eta,u^{(k-1)}(\xi,\eta),u_\xi^{(k-1)}(\xi,\eta)\right) d\xi d\eta, \\ &u_x^{(k)}(x,y) = P_x(x,y) - \frac{1}{\pi}\iint_D G_x(x,y;\xi,\eta) f\left(\xi,\eta,u^{(k-1)}(\xi,\eta),u_\xi^{(k-1)}(\xi,\eta)\right) d\xi d\eta. \end{aligned}\right\} \quad (31)$$

$\max\limits_{0\le y\le 1}|h_2(y) - h_1(y)| = m$ bo'l sin.

(31) da k = 1 deb va (16)-(17) baholardan foydalanib

$$|u^{(1)}| < N + \frac{3\cdot 2m\cdot CM}{2\pi} y^{\frac{2}{3}}, \ |u_x^{(1)}| < N + \frac{4\cdot 2m\cdot CM}{\pi} y^{\frac{1}{4}}.$$

F (x,y,u,ux) funksiya u , ux argumentlari (x, y) yopiq D sohadagi qiymatlari uchun (30) tengzisliklarni qanoatlantirishi uchun, qaysiki bunda f (x, y, u, ux) chegaralgan, quyidagi shartarning bajarishi zarur

$$N + \frac{3mCM}{\pi} y^{\frac{2}{3}} \le N+1, \ N + \frac{8mCM}{\pi} y^{\frac{1}{4}} \le N+1.$$

yoki

$$y \le \left(\frac{\pi}{3mCM}\right)^{\frac{3}{2}}, \ y \le \left(\frac{\pi}{8mCM}\right)^{4}.$$

$$\min\left\{\left(\frac{\pi}{3mCM}\right)^{\frac{3}{2}}; \left(\frac{\pi}{8mCM}\right)^{4}\right\} = y_0. \quad (32)$$

(31) munosabatlardan k=2 uchun quyidagilarga ega bo'lamiz

$$|u^{(2)}| < N + \frac{3mCM}{\pi} y^{\frac{2}{3}}, \ |u_x^{(2)}| < N + \frac{8mCM}{\pi} y^{\frac{1}{4}}.$$

munosabatga ko'ra esa

$$|u^{(2)}| < N+1, \ |u_x^{(2)}| < N+1$$

ya'ni u va ux argumentlarning takroriy (ikkinchi) yaqinlashishida ham f (x,y,u, ux) funksiya (30) chegaralangan sohadan chiqmaydi.

To'liq induksiya usulini qo'llagan holda, agar (32) shart bajarilsa, ketma-ket yaqinlashishlarning hech biri hadi (30) sohani tark etmaydi degan xulosaga kelamiz.

Endi $\left\{\dfrac{\partial^i u^{(n)}}{\partial x^i}\right\} \ (i = 0,1)$ ketma-ketlik limiti mavjudligini ko'rsatamiz.

Buning uchun quyidagi qatorning yaqinlashuvini ko'rsatamiz

$$\frac{\partial^i u^{(0)}}{\partial x^i}+\left(\frac{\partial^i u^{(1)}}{\partial x^i}-\frac{\partial^i u^{(0)}}{\partial x^i}\right)+\left(\frac{\partial^i u^{(2)}}{\partial x^i}-\frac{\partial^i u^{(1)}}{\partial x^i}\right)+\ldots+\left(\frac{\partial^i u^{(n)}}{\partial x^i}-\frac{\partial^i u^{(n-1)}}{\partial x^i}\right)+\ldots \qquad (33)$$

qator hadlarning absolyut qiymatlarini baholaylik

$$\left.\begin{aligned}&\left|u^{(k)}-u^{(k-1)}\right|\le\frac{2^{k-1}m^kC^kL^{k-1}M}{\pi^k}\prod_{q=1}^{k}B\left(\frac{2q+1}{3},\frac{2}{3}\right)y^{\frac{2k}{3}},\\&\left|u_x^{(k)}-u_x^{(k-1)}\right|\le\frac{2^{k-1}m^kC^{k-1}C_1L^{k-1}M}{\pi^k}\prod_{q=1}^{k-1}B\left(\frac{2q+1}{3},\frac{2}{3}\right)B\left(\frac{2k+1}{3};\frac{3}{4}\right)y^{\frac{8k+1}{4}}\end{aligned}\right\}$$

Ko‘rinib turibdiki, (33) qatorlar har bir hadining absolyut qiymati quyidagi darajali qatorlar mos hadlaridan katta emas:

$$\sum_{k=1}^{\infty}\frac{2^{k-1}m^kC^kL^{k-1}M}{\pi^k}\prod_{q=1}^{k}B\left(\frac{2q+1}{3},\frac{2}{3}\right)y^{\frac{2k}{3}},\ \sum_{k=1}^{\infty}\frac{2^{k-1}m^kC^{k-1}C_1L^{k-1}M}{\pi^k}\prod_{q=1}^{k-1}B\left(\frac{2q+1}{3},\frac{2}{3}\right)B\left(\frac{2k+1}{3},\frac{3}{4}\right)y^{\frac{8k+1}{3}} \qquad (34)$$

Bu qatorni yaqinlashishini tekshiramiz. D'Alembert mezonini qo‘llagan holda, quyidagiga ega bo‘lamiz

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right|=\lim_{k\to\infty}\frac{2mCL}{\pi}B\left(\frac{2k+1}{3},\frac{2}{3}\right)=0,\ \lim_{k\to\infty}\left|\frac{b_{k+1}}{b_k}\right|=\lim_{k\to\infty}\frac{2mCL}{\pi}\frac{B\left(\frac{2k-1}{3},\frac{2}{3}\right)B\left(\frac{2k+1}{3},\frac{3}{4}\right)}{B\left(\frac{2k-1}{3},\frac{3}{4}\right)}=0.$$

(31) tenglamada integral ostida limitga o‘tib, quyidagiga ega bo‘lamiz

$$u(x,y)=P(x,y)-\frac{1}{\pi}\iint_D G(x,y;\xi,\eta)f\left(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta)\right)d\xi d\eta\ .$$

Biz yechimni $D_0=\{h_1(y)\le x\le h_2(y),\ 0<y\le y_0\}$ sohada mavjudligini ko‘rsatdik, garchi masala $D=\{(x,y):h_1(y)<x<h_2(y),0<y\le 1\}$ sohada qoyilgan bo‘lsa ham. D1 va D sohalar turlicha bo‘lishi mumkin.

Agar $y_0\ge 1$ bo‘lsa, bu holda masala to‘liq yechilgan hisoblanadi.

Agar y0 <1 bo‘lsa, yechimni davom ettirish mumkin. Buni quyidagicha amalga oshiramiz. Masalani D1 =D\D0 sohada o‘rganamiz.

Berilgan sohadagi masalani yechish uchun yuqoridagi sxema asosida, chiziqli bo‘lmagan Volterra integral tenglamasini hosil qilamiz.

$$u(x,y)=P(x,y)-\frac{1}{\pi}\iint_{D_1} G(x,y;\xi,\eta)f\left(\xi,\eta,u(\xi,\eta),u_\xi(\xi,\eta)\right)d\xi d\eta\ .$$

Yuqoridagi kabi mulohazalarda bu integral tenglamani berilgan sohada ketma-ket yaqinlashish usulida yechish mumkin.

$$D_1=\{h_1(y)\le x\le h_2(y),\ y_0<y\le y_1\}.$$

$$y\le y_0+\left(\frac{\pi}{3CM}\right)^{\frac{3}{2}},\ \ y\le y_0+\left(\frac{\pi}{8mCM}\right)^4$$

bu yerda

$$\min\left\{\left(\frac{\pi}{3mCM}\right)^{\frac{3}{2}};\left(\frac{\pi}{8mCM}\right)^{4}\right\}=y_0,$$

yoki

$$y\le 2y_0.$$

Agar shundan so‘ng ham $y<1$ bo‘lsa, yuqoridagi jarayonni takrorlash mumkin. Har bir qadamda $\delta=y_{k+1}-y_k=y_0$ musbat qadamga yurganimiz uchun va bu qadam oldingisidan kichik emasligidan, chekli sondagi qadamlardan so‘ng D sohani to‘liq qoplash mumkin.

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