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BUTUN SONLI KETMA-KETLIKLARGA DOIR MASALAR YECHISHDA ANT'YE VA MANTISSA XOSSALARIDAN FOYDALANISH

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Annotatsiya: *Ushbu maqolada ant'ye va mantissa mavzusiga doir nazariy va amaliy ma'lumotlar hamda isbotlar algebra va sonlar nazariyasidan keltirilgan.Bir necha tipik hamda olimpiada darajasidagi masalalarni ant'ye va mantissaning xossalaridan foydalangan holda yechish metodlari tavsiya qilingan.*

Kalit so'zlar: *Ant'ye, mantissa,tub son,sonli ketma-ketliki, analitik,recurrent,butun sonli ketma-ketliklar, Fibonachchi ketma-ketligi,Bine formulasi,qo'sh tengsizlik.*

Ushbu maqolada sonli *ketma-ketliklarni* natural sonlar to'plamida aniqlangan haqiqiy qiymatlarni qabul qiluvchi funksiya sifatida qaraymiz.Sonli ketma-ketliklarni $y = f(n)$ va mantissani esa unga o'xshash $\{ y_n \}$ kabi belgilanishidan chalkasmaslik kerak, ularning yozilishidan farqi ko'rinib turibdi.

Sonli tengsizliklarni turli usullarda berish mumkin.Shulardan asosiylarini sanab o'tamiz: analitik , rekurrent,bevosita (verbal) va dastlabki bir nechta hadlarini berish orqali.Quyida ularning ayrimlariga izoh berib o'tamiz.

Ketma-ketlik *analitik* usulda berilganda n- hadi formulasi orqali beriladi. Masalan, $a_n = \frac{n(n+1)}{2}$.

Ketma-ketlikning *rekurrent* usulda berilishining mohiyati shundaki, har bir keying hadi oldingi hadi orqali ifodalanadi.Rekurrent so'zining ma'nosi lotincha " recurro" sozidan olingan bo'lib, "qaytish" degan ma'noni bildirar ekan.Masalan, mashhur Fibonachchi[1] sonlari ketma- ketligini keltirsak bo'ladi:

$$F_1 = 1, F_2 = 1 , F_{n+2} = F_{n+1} + F_n \ (n \epsilon N).$$

Ketma-ketlik *dastlabki bir nechta hadlari* orqali berilganda, odatda uning dastlabki 10-15 hadi berilgandagi qonuniyat orqali qolgan hadlarini ham yozish mumkin bo'ladi va usulda ketma-ketliklarni tuzish *bevosita(verbal)* ketma-ketliklarning berilish usulidan ko'rgazmaliroq va tushinarliroq bo'ladi.Masalan,

$$a_n = \{ 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, \dots \} .$$

Yuqoridagi ketma-ketlik shunday natural sonlar ketma-ketligidan iboratki, undan to'la kvadrat bo'ladigan natural sonlar chiqarib tashlangan.

Shuni aytib o'tish kerakki maktab matematika kursida arifmetik va geometrik progressiyalarni o'rganishga yetarlicha vaqt ajratilgan.Biz quyida maktab matematika kursida uchramaydigan ant'ye va mantissa xossalari qo'llaniladigan arifmetik va geometrik progressiyaga doir ikkita masala va mavzuga doir ta masalani ko'rib o'tamiz.

1. Ayirmasi nolga teng bo'lmagan arifmetik progressiya tashkil qiluvchi uchta haddan iborat x, $[x]$, va $\{x\}$ larning qiymatlarini toping.

Yechilishi. Quyidagi belgilashlarni kiritib olamiz**.**

$x = n + \alpha$, $[x] = n$,$\{n\} = \alpha$. U holda $\alpha_1 = n + \alpha$,$\alpha_2 = n$ va $\alpha_3 = \alpha$ sonlar arifmetik progressiyani tashkil qiladi $\{\alpha_1, \alpha_2, \alpha_3\}$,bunda $\alpha_2 = \frac{\alpha_1+\alpha_3}{2}$. [1] [1]Fibonachchi (taxallus), L. Pizanskiy (L.Pizano , taxminan 1170 – 1250 yillar) – italiyalik matematik , o'rta asrlar Evropasining yirik matematigi.

o'lishi mumkin bo'lgan hollar soni uchta :

1) $n = \frac{\alpha+(n+\alpha)}{2}$, 2) $\alpha = \frac{n+(n+\alpha)}{2}$, 3) $n + \alpha = \frac{\alpha+n}{2}$,bunda $n \neq 0$ va $\alpha \neq 0$, chunki shartga ko'ra progressiyaning ayirmasi nolga teng emas.

Birinchi holni ko'rib chiqamiz. $n = \frac{\alpha+(n+\alpha)}{2}$,bunda, $n \epsilon Z_{\neq 0}$, $0 < \alpha < 1$,

tenlamaning yechimi $n = 1$, $\alpha = \frac{1}{2}$ bo'ladi. U holda ayirmasi $d = \frac{1}{2}$ ga teng bo'lgan bizdan so'ralgan arifmetik progressiyaning uchta hadi

$\alpha_1 = \{x\} = \frac{1}{2}$, $\alpha_2 = [x] = 1$, $\alpha_3 = x = \frac{3}{2}$. bo'ladi.

Ikkinchi va uchinchi hollarda yechim yo'q (O'ylanib ko'ring) .

Javob: $x = \frac{3}{2}$, $[x] = 1$, $\{x\} = \frac{1}{2}$.

2. (Kanada /1975 masalasi asosida) Maxraji nolga teng bo'lmagan geometrik progressiya tashkil qiluvchi uchta haddan iborat x, $[x]$, va $\{x\}$ larning qiymatlarini toping (Masalaning original variantida $x > 0$).

Yechilishi. Quyidagi belgilashlarni kiritib olamiz**.**

$x = n + \alpha$, $[x] = n$,$\{n\} = \alpha$. U holda $b_1 = n + \alpha$,$b_2 = n$ va $b_3 = \alpha$ sonlar geometrik progressiyani tashkil qiladi $\{ b_1, b_2, b_3$, $bunda\ b^2 = b_1 \cdot b_3$.

Quyidagi uchta hol bo'lishi mumkin:

1) $\alpha^2 = n(n + \alpha)$, 2) $n^2 = \alpha(n + \alpha)$, 3) $(n + \alpha)^2 = n\alpha$,bunda

$n \neq 0$ va $\alpha \neq 0$, masala shartiga ko;ra progressiyaning maxraji nolga teng emas.

Birinchi holni ko'rib chiqamiz. $\alpha^2 = n(n+\alpha)$, bunda $n \epsilon Z_{\neq 0}$, $0 < \alpha < 1$,

tenglamaning yechimi $n = -1$, $\alpha = \frac{\sqrt{5}-1}{2}$ bo'ladi. U holda maxraji $q = \frac{1-\sqrt{5}}{2}$ ga teng bo'lgan geometrik progressiya ushbu

$$b_1 = [\,x] = -1\,, b_2 = \{\,x\,\} = \frac{\sqrt{5}-1}{2}\,, \qquad b_3 = x = \frac{\sqrt{5}-3}{2}.$$

ko'rinishda bo'ladi.

kkinchi holda maxragi $q = \frac{\sqrt{5}+1}{2}$ ga teng bo'lgan geometrik progressiya hosil bo'ladi:

$$b_1 = \{\,x\,\} = \frac{\sqrt{5}-1}{2}\,, b_2 = [\,x\,] = 1\,, \qquad b_3 = x = \frac{\sqrt{5}+1}{2}.$$

Uchinchi holda yechim yo'q.

Javob: $\left[\begin{array}{l} 1)\ x_1 = \frac{\sqrt{5}-3}{2}\ , [\,x_1\,] = -1\,,\ \{x_1\} = \frac{\sqrt{5}-1}{2} \\ 2)\ x_2 = \frac{\sqrt{5}+1}{2}\ , [\,x_2] = 1\,,\ \{x_2\} = \frac{\sqrt{5}-1}{2} \end{array}\right.$.

3. Fibonachchi sonlari ketma-ketligi uchbu

$$F_n = \left[\frac{\varphi^n}{\sqrt{5}} + \frac{1}{2}\right], \text{bunda } \ \varphi = \frac{1+\sqrt{5}}{2}, n\,\epsilon\,\mathbb{N}.$$

formula yordamida hisoblanishini isbotlang.

Isboti. Bine [2] formulasidan foydalanamiz

$F_n = \frac{\varphi^n - \varphi_1}{\sqrt{5}}$, bunda $\varphi_1 = \frac{1-\sqrt{5}}{2}$.

Endi $n \epsilon N$ larda $-\frac{1}{2} < \frac{\varphi_1}{\sqrt{5}} < \frac{1}{2}$ ekanidan quyidagilarga ega bo'lamiz.

$\frac{\varphi^n}{\sqrt{5}} - \frac{1}{2} < F_n < \frac{\varphi^n}{\sqrt{5}} + \frac{1}{2}$, $\quad F_n - \frac{1}{2} < \frac{\varphi^n}{\sqrt{5}} < F_n + \frac{1}{2}$,

$$F_n < \frac{\varphi^n}{\sqrt{5}} + \frac{1}{2} < F_n + 1.$$

Demak, $F_n = \left[\frac{\varphi^n}{\sqrt{5}} + \frac{1}{2}\right]$. Isbot tugadi.

4. 0 va 1 dan tashkil topgan sonli ketma-ketlik berilgan:

$$\{a_n\} = \{1, 0,0, 1, 0, 0, 0, 0, 1, 0, 0, \ldots \ \} = \begin{cases} 1, \text{agar } \ n \ \text{ to'la kvadrat bo'lsa,} \\ \qquad\qquad \text{aks holda, } 0 \end{cases}$$

Yuqoridagi ketma-ketlikning n – hadi formulasini keltirib chiqaring.

Yechilishi. Masalaning yechish g'oyasi quyidagi xossaga asoslanadi

$$\{\sqrt{n}\} = \begin{cases} 0, & \text{agar } \exists k \in \mathbb{N} : n = k^2 \\ \alpha, & \text{agar } (0 < \alpha < 1). \end{cases}$$

[2]J.Ph.M.Binet — fransuz matematigi, mexanik va astronom.Biroq Bine formulasini undan oldin kelib chiqishi fransuz bo'lgan angliyalik matematik A.Muavr (A.Moivre, 1667-1754 y.) keltirib chiqargan.

n to'la kvadrat bo'lganda yechim quyidagicha bo'ladi

$$1 - \{\sqrt{n}\} = \begin{cases} 1, & \text{agar } \exists k \in \mathbb{N} : n = k^2 \\ \alpha, & \text{agar } (0 < \alpha < 1). \end{cases}$$

Javob: $a_n = [\ 1 - \ \{\sqrt{n}\}, n \in \mathbb{N}$.

Eslatma . 5- misoldagi shartlarda n- hadini hisoblashning boshqa formulasi ham mavjud ekan. O'zingiz mustaqil o'ylanib ko'ring.

5. 0 va 1 dan tashkil topgan sonli ketma-ketlik berilgan:

$$\{a_n\} = \{1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, \dots\} = \begin{cases} 1, \text{agar n } - \text{uchburchakli son bo'lsa,} \\ \qquad \text{aks holda, } 0 \end{cases}$$

Yuqoridagi ketma-ketlikning n – hadi formulasini keltirib chiqaring.

Yechilishi. k - orindagi uchburchakli sonlarni topish formulasidan

$T_k = \frac{k(k+1)}{2} : k = \frac{\sqrt{8T_k+1}-1}{2}$ hamda oldingi (5- misol) misoldagidek uchbu

$$1 - \{\sqrt{k}\} = \begin{cases} 1, & \text{agar } \exists k \in \mathbb{N} : n = \frac{k(k+1)}{2} \\ \alpha, & \text{agar } (0 < \alpha < 1). \end{cases}$$ ifodani hosil qilish mumkin.

Javob: $a_n = \left[1 - \left\{\frac{\sqrt{8n+1}-1}{2}\right\}\right], \ n \in \mathbb{N}.$

6. Quyidagi shartlar asosida berilgan kamaymaydigan natural sonlar ketma-ketkigining n – hadi formulasini keltirib chiqaring.

$$\{a_n\} = \{1, 2, 3, 5, 7, 10, 13, 17, 21, 26, 31, 37, \dots \},$$

$a_{2m} - a_{2m-1} = a_{2m+1} - a_{2m} = m, \ m \in \mathbb{N}.$

Yechilishi. $\{a_n\}$ ketma-ketlik uchun ushbu

$$a_{1,} = 1, \qquad a_{n+1} = a_n + \left[\frac{n+1}{2}\right], n \in \mathbb{N}$$

rekurrent formulani keltirib chiqarish mumkin.

U holda

$$a_n = \left[\frac{n}{2}\right] + a_{n-1} = \left[\frac{n}{2}\right] + \left[\frac{n-1}{2}\right] + \dots + \left[\frac{3}{2}\right] + \left[\frac{2}{2}\right] + a_1.$$

n toq va juft son bo'lgan hollarni ko'rib chiqamiz. Oldin **n toq son** bo'lgan holni ko'rib o'tamiz.

$$a_n = \frac{n-1}{2} + \frac{n-1}{2} + \left(\frac{n-1}{2} - 1\right) + \left(\frac{n-1}{2} - 1\right) + \dots + 1 + 1 + a_1 =$$

$$= \frac{n-1}{2} \cdot \frac{n-1}{2} + 1 = \left[\frac{n}{2}\right] \cdot \left[\frac{n+1}{2}\right] + 1.$$

Endi **n juft son** bo'lgan holni ko'rib chiqamiz. U holda

$$a_n = \frac{n}{2} + \left(\frac{n}{2} - 1\right) + \left(\frac{n}{2} - 1\right) + \dots + 1 + 1 + a_1 =$$

$$= \frac{n}{2} \cdot \frac{n}{2} + 1 = \left[\frac{n}{2}\right] \cdot \left[\frac{n+1}{2}\right] + 1.$$

Javob: 1) Rekkurent formulasi $a_1 = 1,\ a_{n+1} = a_n + \left[\frac{n+1}{2}\right], n \in \mathbb{N}$.

2) n- hadining formulasi $a_n = 1 + \left[\frac{n}{2}\right] \cdot \left[\frac{n+1}{2}\right]$.

7. m soni roppa-rosa 2m marta uchraydigan

$$\{a_n\} = \left\{1,2,2,3,3,3, \dots, \underbrace{m, m, \dots, m}_{2m\ ta\ element}\right\}.$$

kamaymaydigan natural sonlar ketma-ketligi berilgan. Shu ketma-ketlikning n – elementini aniqlovchi formulani keltirib chiqaring.

Yechilishi. n –hadini keltirib chiqarish usuli (oldindi) masalanikiga o'xshashligidan yechish bosqichlariga izoh bermaymiz.

$$2 + 4 + \dots + 2(m-1) \le n \le 2 + 4 + \dots + 2m,$$

$$2 \cdot \frac{m(m-1)}{2} + 1 \le n \le 2 \cdot \frac{m(m+1)}{2},$$

$$\left(m - \frac{1}{2}\right)^2 < m^2 - m + 1 \le n \le m^2 + m < \left(m + \frac{1}{2}\right)^2,$$

Demak, $m = a_n = \left[\sqrt{n} + \frac{1}{2}\right], n \in \mathbb{N}$.

Javob : $a_n = \left[\sqrt{n} + \frac{1}{2}\right]$, $n \in \mathbb{N}$.

8. (NMMT/2003) $\{a_n\}$ sonli ketma-ketlik quyidagicha berilgan:

$$a_1 = 1, a_n = \left[\frac{n^3}{a_{n-1}}\right], n \in \mathbb{N}_{\ge 2}\ .\ a_{999} \text{ ni toping.}$$

Yechilishi. Ketma-ketlikning dastlabki bir nechta hadlarini hisoblaymiz:

$a_1 = 1, a_2 = 8, a_3 = 3, a_4 = 21, a_5 = 5, a_6 = 43, a_7 = 7.$

Yuqoridagi hisoblashlardan ketma-ketlikning toq indeksli hadlarining qiymatlari o'zining indeksi qiymatiga tengligini ko'rish mumkin.Shuni isbotlaymiz.

$a_n = n$ va n toq son bo'lsin. U holda

$$a_{n+1}=\left[\frac{(n+1)^3}{a_n}\right]=\left[\frac{(n+1)^3}{n}\right]=n^2+3n+3,$$

$$a_{n+2}=\left[\frac{(n+2)^3}{a_{n+1}}\right]=\left[\frac{n^3+6n^2+12n+8}{n^2+3n+3}\right]=$$

$$n+\left[\frac{3n^2+9n+8}{n^2+3n+3}\right]=n+3+\left[-\frac{1}{n^2+3n+3}\right]=n+2.$$

Javob: $a_{999}=999$

9. Yig'indini hisoblang.

$$\left[\frac{1}{3}\right]+\left[\frac{2}{3}\right]+\left[\frac{2^2}{3}\right]+\left[\frac{2^3}{3}\right]+\cdots+\left[\frac{2^{1000}}{3}\right].\ \ (\text{A. Golovanov})$$

Yechilishi. Yig'indidagi birinchi qo'shiluvchini chetga surib qo'yib (nolga tengligi uchun), qolgan 1000 ta qo'shiluvchining yig'indisini qaraymiz

$$\frac{2}{3}+\frac{2^2}{3}+\frac{2^3}{3}+\cdots+\frac{2^{1000}}{3}.$$

Yuqoridagi yig'indi geometrik progressiyaning yig'indisi va u $\left[\frac{2^{1001}-2}{3}\right]$ ga teng.

Endi qo'shiluvchilarning har birini butun qismlari bilan almashtiramiz.Yi'g'indining hech qasi biri butun son emasligiga e'tiborni qaratamiz va ixtiyoriy ketma-ket hadlariniong yi'gindisi esa butun son (chunki, $\frac{2^k}{3}+\frac{2^{k+1}}{3}=\frac{3\cdot 2^k}{3}=2^k$). Ma'lumki, butun bo'lmagan ikkita sonning yig'indisi butun son bo'lsa, ularning butun qismlari yig'indisi o'zlarining yig'indisidan bittaga kam bo'ladi ($[\alpha+\beta]$=$[\{\alpha\}+\{\alpha\}+[\beta]+\{\beta\}]=[\alpha]+[\beta]+[\{\alpha\}+\{\beta\}]=$

$$=[\alpha]+[\beta]+[1]=[\alpha]+[\beta]+1).$$

Shuning uchun biz qarayotgan geometrik progressiyada har bir ikkita ketma-ket hadlarini yig'indisini butun qismlari yi'indisi bilan almashtirganda yi'gindi 1 taga kamayib, umumiy yig'indi 500 ga kamayadi,chunki qo'shiluvchilar soni 1000 ta.

Javob: $\frac{2^{1001}-2}{3}-500.$

10. Ixtiyoriy natural n lar uchun quyidagi tengsizlikni isbotlang.

$$\sum_{k=1}^{n^2}\{\sqrt{k}\}\le\frac{n^2-1}{2}$$

(bunda , $\{k\}-k$ ning kasr qismi).

Isboti. $n=1$ da tengsizlik $0=0$ tenglikka aylanadi. $n>1$ da kasr qismlar yig'indisi ikkita ketma-ket kelgan kvadratlar orasida quyidagi tengsizlikni qanoatlantirishini isbotlaymiz

$$\sum_{k=m^2}^{m^2+2m} \{ k \} \le \frac{2m+1}{2}. \qquad (1)$$

O'rta arifmertik va o'rta geometrik qiymatlar to'g'risidagi tengsizlikka ko'ra quyidagilarni yozib olamiz

$$\sqrt{m^2+a}+\sqrt{m^2+2m-a} \le \sqrt{2(2m^2+2m)} < 2m+1 \ , 0 \le a \le m$$

Yuqoridagi munosabatdan

$$\left\{\sqrt{m^2+a}\right\}+\left\{\sqrt{m^2+2m-a}\right\} \le 1. \qquad (2)$$

kelib chiqadi.Bu tengsizliklarni ($a=0,1,\ldots,m-1$ larda) va $\{\sqrt{m^2+m}\} \le \frac{1}{2}$

qo'shib (hosil bo'lganni (2) ni $a=m$ da 2 la qismini 2 ga bo'lib), (1) ga kelamiz.

(1) da yig'indini m ni 1 dan $n-1$ gacha hisoblab, quyidagini hosil qilamiz.

$$\sum_{k=1}^{n^2-1} \{ k \} \le \frac{n^2-1}{2}.$$

Endi , $\{ \sqrt{n^2} \} = 0$ ekanligini hisobga olsak bo'ldi.Isbot tugadi.

Mustaqil ishlash uchun masalalar

1. (Abstriya-Polsha 1993) $\{a_n\}$ sonli ketma-ketlik quyidagicha rekurrent formula orqali berilgan

$$a_{n+1} = \left[\sqrt[3]{a_n+n}\right]^3, a_1 = 0 , n \in \mathbb{N}.$$

a) n- hadi formulasini keltirib chiqaring;

b) $a_n = n$ bo'ladigan n ning barcha qiymatlarini toping.

2. (AQSH / Gerald Heuer) O'suvchi

$\{a_n\} = \{1, 2, 4, 5, 7, 9, 10, 12, 14, 16, \ldots\}$ natural sonlar ketma-ketligi oldin bitta toq son, keyin ikkita juft son ,keyin uchta toq son va hakozo qonuniyat asosida berilgan.Shu ketma-kerlikning n – hadi formulasini keltirib chiqaring.

3. (Germaniya/1971-1972) Natural sonlar ketma-ketligidan iborat sonli ketma-ketlik n- hadi formulasi orqali berilgan

$$a_n = n + \left[\sqrt{n} + \frac{1}{2}\right] , n \in \mathbb{N}$$

Shu ketma-ketlikning dastlabki yigirma hadini toping.

4. (JAR/2005) O'suvchi natural sonlardan iborat ketma-ketlik berilgan**.**

$\{a_n\} = \{1, 2, 4, 5, 7, 9, 10, 12, 14, 16, \dots\}$.

Shu ketma-ketlik n – hadi formulasini keltirib chiqaring.

5. (Nordic/ 2013) $\{ a_n \}$ Sonli ketma-ketlik ushbu

$$a_1 = 1, \qquad a_{n+1} = a_n + \left[\sqrt{a_n} + \frac{1}{2} \right], \qquad n \epsilon \mathbb{N}$$

rekurrent formula bilan berilgan. $n < 2013$ shart bajarilganda to'la kvadrat bo'ladigan n larni toping.

6. (Bolgariya/1996) Sonli ketma-ketlik ushbu

$$a_1 = 1, a_{n+1} = \frac{a_n}{n} + \frac{n}{a_n}, n \epsilon \mathbb{N}$$

rekurrent formula bilan berilgan. $n \geq 4$ larda $[\, a_n^2 \,] = n$ tenlik o'rinli bo'lishini isbotlang.

7. (IMO/1985) $\{\boldsymbol{a_n}\}$ sonli kema-ketlikda , bunda $\boldsymbol{a_n} = \left\{ 2^n \sqrt{2} \right\}$, $n \epsilon \mathbb{N}$ $\frac{1}{2}$ dan katta cheksiz ko'p element borligini isbotlang.

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