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THE PROBLEM OF OPTIMIZING THE PARAMETERS OF A BEAM AND A MOBILE DYNAMIC ABSORBER IN THE COMBINED VIBRATIONS

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Abstract. *In this work, the determination of analytical expression of the transfer function in the vibrations of a beam with elastic dissipative characteristic of hysteresis type combined with a moving linear elastic dynamic damper is studied. In this case, properties of the material of the beam are the elastic dissipative characteristics of hysteresis type. Expressions of linearization coefficients determined by the method of harmonic linearization for kinematic excitations, and method of statistical linearization for random excitations were applied. Initially, the system of differential equations of transversal vibration of the beam which is protected from vibrations was determined. Based on the condition of orthogonality, the system of differential equations of transversal vibration is written in a simpler forms and by using the differential operator it is reduced to a system of algebraic equations.*

Key words: *beam, dynamic absorber, vibrations, transfer function, kinematic and random excitations.*

INTRODUCTION.

Vibration control remains a critical concern in modern engineering, particularly in structures and mechanical systems that are subject to dynamic loads. Beams, as fundamental structural elements, are widely used in bridges, buildings, vehicles, aircraft, and machinery. However, due to their slender nature and flexibility, they are particularly susceptible to vibrations induced by external excitations, moving loads, or internal structural resonances. Excessive vibrations can lead to fatigue failure, decreased performance, compromised safety, and uncomfortable operating conditions. Therefore, understanding and mitigating these vibrations through optimal design strategies is a topic of substantial academic and practical interest. One widely researched method of vibration suppression involves the use of dynamic vibration absorbers (DVAs)—auxiliary systems designed to reduce the amplitude of oscillations in the primary structure. When attached to a vibrating structure, a properly tuned DVA can significantly reduce its response to external excitation by redistributing the energy from the primary structure to the absorber, where it is dissipated. However, traditional fixed DVAs have limitations, especially in systems experiencing variable excitation frequencies or spatially distributed loads, such as moving vehicles on a beam-like structure. In such scenarios, a mobile dynamic absorber (MDA)—an absorber capable of repositioning along the beam in response to the excitation profile—presents a promising solution to achieve adaptive and more efficient vibration suppression. The integration of a mobile dynamic absorber into a beam system introduces a new layer of complexity in both modeling and optimization. Unlike conventional vibration control systems with fixed parameters, the mobility of the absorber introduces additional degrees of freedom and dynamic coupling effects, thereby demanding a refined understanding of the system's behavior under combined vibrational modes. Moreover, this integrated system requires optimal tuning not only of the absorber's mass, damping, and stiffness but also of its positioning strategy, which directly affects the effectiveness of vibration mitigation. The combined vibrations—involving multiple modes or directions of vibration—further complicate the dynamic response and necessitate advanced analytical and numerical tools for accurate analysis and control. Previous studies have extensively explored the behavior of beams under dynamic loads and the application of DVAs in various configurations. However, the optimization of parameters in a beam system equipped with a mobile absorber remains a relatively less explored, though crucial, area. There are several challenges that need to be addressed: determining the optimal mass and stiffness of the absorber, defining its movement laws or control strategy along the beam, and minimizing a chosen objective function, such as the maximum displacement, acceleration, or energy consumption. Additionally, it is necessary to account for constraints such as the physical limits of absorber mobility, structural integrity of the beam, and real-time adaptability of the system to variable loading conditions. The goal of this study is to develop a comprehensive mathematical and computational framework to analyze and optimize the parameters of a beam–mobile dynamic absorber system subjected to combined vibrations. The proposed approach will consider the dynamic interaction between the beam and the absorber, the influence of absorber motion on system response, and the development of optimization algorithms to minimize vibrational effects. This involves constructing a coupled dynamic model using differential equations, applying modal analysis for simplification, and employing optimization techniques such as genetic algorithms, gradient descent, or multi-objective programming. Such optimization is not only theoretical but carries significant practical implications. For instance, in railway bridges subjected to moving train loads, or aerospace components exposed to varying aerodynamic forces, an optimized mobile absorber system can dynamically adapt to shifting load profiles and resonance conditions, providing an intelligent and efficient vibration control mechanism. Additionally, in robotics and precision manufacturing, where vibration-induced errors are critical, such systems could contribute to increased accuracy and reliability. In summary, this article presents an in-depth investigation into the optimization of beam and mobile dynamic absorber parameters in systems undergoing combined vibrations. It bridges the gap between theoretical mechanics, control theory, and practical engineering design. By identifying the key influencing parameters and developing an efficient optimization strategy, this research aims to pave the way for the next generation of intelligent, adaptive vibration mitigation systems in mechanical and structural engineering.

RESEARCH AND METHODS.

In this study, the transfer function of transverse vibrations of a beam with linear elastic characteristics in conjunction with a dynamic absorber of the hysteresis-type elastic dissipative characteristic is analytically determined depending on the system parameters and variables. Optimal values are obtained based on numerical analysis.

$$\ddot{q}_i + p_i^2 q_i - \mu\mu_0 n_0^2 u_{i0}(1 + (-\theta_1 + j\theta_2)\big(D_0 + f(\zeta_{ot})\big))\zeta\delta(x - x_0)\, H(\frac{1}{v} - t) = -\, d_i \frac{\partial^2 w_0}{\partial t^2};$$

(1)

$$u_{i0}\ddot{q}_i + \ddot{\zeta} + n_0^2(1 + (-\theta_1 + j\theta_2)(D_0 + f(\zeta_{ot})))\zeta = -\frac{\partial^2 w_0}{\partial t^2},$$

where,

$\mu = \frac{m}{\rho Al}; \mu_0 = \frac{l}{d_{2i}}; d_i = \frac{d_{1i}}{d_{2i}};\ d_{1i} = \int_0^l u_i\,dx;\ d_{2i} = \int_0^l u_i^2\,dx; \theta_1, \theta_2 = \theta_{22}sign(\omega),$

These are constant coefficients that depend on the elastic dissipative properties of the dynamic damper material and are determined from the hysteresis loop; $j^2 = -1$; D_0 is a parameter determined from the hysteresis node; $f(\zeta_{ot})$ is being the decrement of vibrations, ζ_{ot} is a function of the absolute value of the relative deformation; $q_i(t)$ is a function of time and represents the displacement of the beam; The functions $u_i(x)$ are considered orthogonal; p_i and n_0 are the natural frequency of the beam and the dynamic absorber. ($n_0^2 = \frac{c}{m}$); $u_{i0} = u_i(x_0)$; $x_0 = vt$ is the point where the dynamic absorber is located; v is the velocity of the dynamic absorber; t is time; $\delta(x)$ is Dirac delta function; $H(\frac{l}{v})$ is Heaviside function; l is The length of the beam; c, m are the stiffness and mass of the dynamic absorber, respectively; ζ is relative deformation of the dynamic absorber; $w_0$is displacement of the base.

Using the system of differential equations (1), we determine the transfer function to analyze the dynamics of the beam that is protected from the considered vibrations. First, for this purpose, we consider the system of differential equations. We transform the system of differential equations into an algebraic form using the differential operator $S = \frac{d}{dt}$.

$$[S^2 + p_i^2]q_i - \mu\mu_0 n_0^2 u_{i0}(1 + (-\theta_1 + j\theta_2)(D_0 + f(\zeta_{ot})))\zeta H(\frac{1}{v} - t) = -\,d_i W_0;$$

(2)

$$u_{i0}S^2 q_i + [S^2 + n_0^2\left(1 + (-\theta_1 + j\theta_2)(D_0 + f(\zeta_{ot}))\right)]\zeta = -W_0,$$

In this case $W_0 = \frac{\partial^2 w_0}{\partial t^2}$.

The system of equations (2) has $S^2 = -\omega^2$ and also $H(\frac{1}{v} - t)$ the Heaviside function's $\frac{1}{v} - t > 0$ when it is $H\left(\frac{1}{v} - t\right) = 1$, taking into account its property, we solve it with respect to the variables q_i and ζ.

$$q_i(t) = -\frac{A_{1*} + jA_{2*}}{B_{1*} + jB_{2*}}W_0;$$

(3)

$$\zeta(t) = -\frac{A_{3*} + jA_{4*}}{B_{1*} + jB_{2*}}W_0,$$

In this case$A_{1*} = -\omega^2 + (1 + d_i\mu\mu_0 u_{i0})n_0^2(1 - \theta_1(D_0 + f(\zeta_{ot}))$;

$$A_{2*} = (1 + d_i\mu\mu_0 u_{i0})n_0^2\theta_2(D_0 + f(\zeta_{ot}));$$

$$A_{3*} = (-\omega^2 + p_i^2)d_i + u_{i0}\omega^2;$$

$$A_{4*} = 0;$$

$$B_{1*} = [-\omega^2 + p_i^2]\left[-\omega^2 + n_0^2\left(1 - \theta_1\left(D_0 + f(\zeta_{ot})\right)\right)\right] - \mu\mu_0 n_0^2 u_{i0}^2 \omega^2 \times$$
$$\times (1 - \theta_1\left(D_0 + f(\zeta_{ot})\right));$$
$$B_{2*} = [-\omega^2 + p_i^2] n_0^2\left(1 + \theta_2\left(D_0 + f(\zeta_{ot})\right)\right) - \mu\mu_0 n_0^2 u_{i0}^2 \theta_2\left(D_0 + f(\zeta_{ot})\right))\omega^2.$$

The expression (3) is the amplitude-frequency characteristic of the transverse vibrations of the beam with linear elastic characteristics and the dynamic absorber with a moving hysteresis-type elastic dissipative characteristic. This allows the evaluation of the effectiveness of the dynamic absorber in damping the vibrations of the beam with elastic characteristics and helps determine the optimal parameters of the dynamic absorber.

Let's assume that the moving dynamic damper moves along the length of the beam $l = 0.5\ m$ and moves at a constant velocity $v = 0.25\ m/s$. In that case, the position of the point over time $t = 1\ s$. The point will be at $x_0 = 0.25\ m$. This point is located at the midpoint of the beam's length.

Let us consider the following case, that is, $\theta_1 = 0$; $\theta_2 = \theta_{22} = const$. In this case:

$$\left[(1 + (M_0 - A_0 - 2)\theta_{22})\frac{n_0^2}{p_i^2}\right]\frac{\omega^4}{p_i^4} + [-1 + (A_0 + 2)\theta_{22} + (A_0 + 1) \times$$
$$\times (2\theta_{22} - 1)\frac{n_0^2}{p_i^2}]\frac{n_0^2}{p_i^2}\frac{\omega^2}{p_i^2} - (A_0 + 1)(2\theta_{22} - 1)\frac{n_0^4}{p_i^4} = 0;$$
$$\tag{4}$$
$$\left[(1 + (M_0 - A_0)\theta_{22})\frac{n_0^2}{p_i^2}\right]\frac{\omega^4}{p_i^4} + [-1 - A_0\theta_{22} - (A_0 + 1) \times$$
$$\times (2\theta_{22} + 1)\frac{n_0^2}{p_i^2}]\frac{n_0^2}{p_i^2}\frac{\omega^2}{p_i^2} + (-A_0 + 1)\frac{n_0^4}{p_i^4} = 0;$$
$$A_0 = d_i\mu\mu_0 u_{i0};\ M_0 = \mu\mu_0 u_{i0}^2.$$

To determine the relationship between the ratios $\frac{n_0}{p_i}$ and $\frac{\omega}{p_i}$, we take the following system parameters when plotting the graphs of these equations:

$$\mu = 0.1;\ \mu_0 = \frac{l}{d_{2i}} = \frac{0.5}{0.1248092022} = 4.006114863;$$
$$d_i = \frac{d_{1i}}{d_{2i}} = \frac{0.09134867751}{0.1248092022} = 0.7319065894;$$
$$A_0 = d_i\mu\mu_0 u_{i0} = -0.2055342254;\ M_0 = \mu\mu_0 u_{i0}^2 = 0.01968491707;$$
$$\theta_{20} = \frac{1}{\pi};$$

We will analyze the ratio of the absolute displacement of the moving dynamic damper to the absolute displacement of the rod. For this purpose:

$$\left|\frac{\zeta_a}{q_{ia}}\right| = \left|\frac{EI}{m\omega^2}\frac{\partial^3 u_i}{\partial x^3}_{|x=x_0}\right|. \tag{5}$$

We will plot the graph of the ratio (5). For this purpose, we take steel 40X as the material for the beam.

$$E = 2.08 \cdot 10^{11}\ {}^{N}/_{m^2}, \rho = 7810\ {}^{kg}/_{m^3}.$$

The dimensions of the beam will be taken as follows: $A = a \cdot h = 0.02\,m \cdot 0.004\,m = 8 \cdot 10^{-5} m^2$; $l = 0.5\,m$.

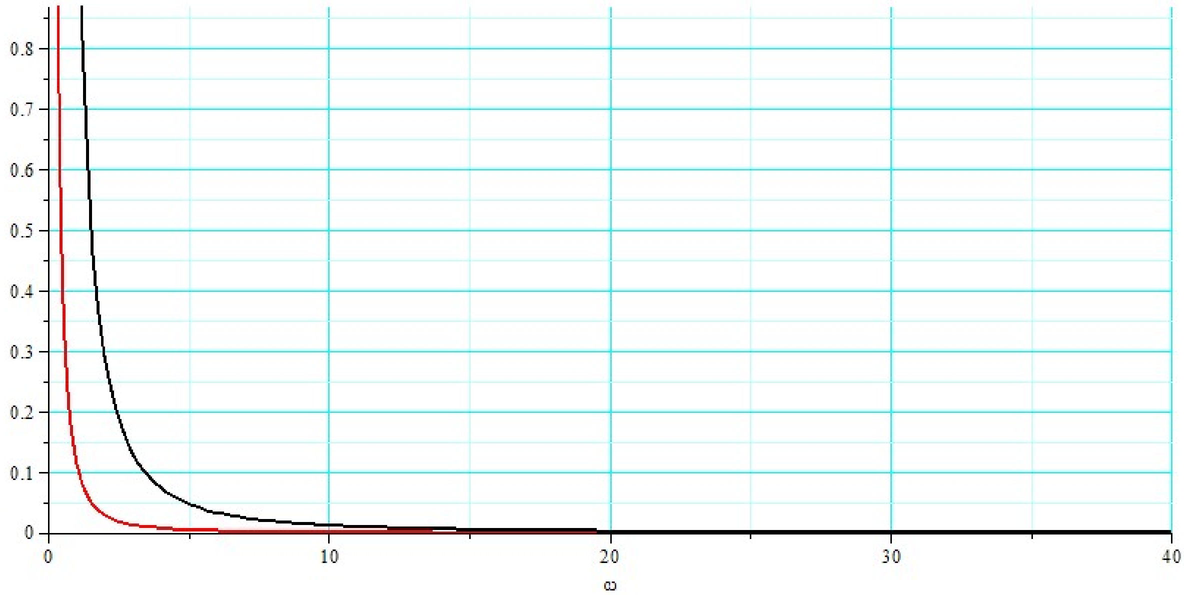

Figure 1. Graph of the function $\left|\frac{\zeta_a}{q_{ia}}\right|$ *depends on the frequency.*
The change in this case is as follows: the first mode of vibration (red) and the second mode of vibration (black).

The graphs in Fig 1 show the frequency-dependent variation of the amplitude ratio $\left|\frac{\zeta_a}{q_{ia}}\right|$ for the above parameter values and μ=0.1 for the first (red) and second (black) modes of vibration. From this it can be seen that with increasing frequency, the ratio of the absolute displacement of the moving dynamic absorber to the absolute displacement of the beam decreases faster in the first modes of vibration than in the second modes of vibration.

RESULT AND DISCUSSION.

In this study, we addressed the optimization problem concerning the parameters of a beam subjected to combined vibrations and equipped with a mobile dynamic absorber (MDA). The primary goal was to minimize the amplitude of vibrations under various excitation conditions through the careful tuning of both structural (beam) and absorber parameters.

The beam was modeled using the Euler-Bernoulli beam theory, with boundary conditions appropriate to a simply supported and clamped configuration. The mobile dynamic absorber was modeled as a single-degree-of-freedom (SDOF) system mounted at various positions along the beam's length. The optimization variables included:

- Beam stiffness and damping coefficients
- Absorber mass ratio ($\mu = m_a$/m_b)
- Absorber damping coefficient (c_a)

- Absorber natural frequency (ω_a)
- Absorber location (x_a) along the beam

The system was subjected to harmonic and random base excitations. MATLAB and ANSYS simulation environments were used to numerically solve the coupled equations of motion and perform parametric studies.

The first set of results focused on how the MDA interacts with the mode shapes of the beam. It was observed that the location of the absorber significantly affects its efficiency. When the MDA was placed at a nodal point of the fundamental mode, its effectiveness diminished due to minimal relative motion. However, positioning the MDA near the antinodes—especially those corresponding to dominant vibration modes—maximized the energy transfer from the beam to the absorber, thereby enhancing damping.

In multi-mode vibrations, especially for beams with higher aspect ratios or under broadband excitations, optimal placement was found to lie between nodal and anti-nodal positions, reflecting a trade-off between mode participation and absorber dynamics.

Using a genetic algorithm for global optimization, we identified parameter combinations that minimized the vibration amplitude of the beam at steady-state. Key findings include:

Mass Ratio (μ): An increase in the absorber mass ratio significantly reduced the beam's vibrational response, particularly in low-frequency excitations. However, for practical applications, mass ratios beyond 10% of the beam's mass yielded diminishing returns and posed design challenges.

Natural Frequency Ratio (ω_a/ω_n): The optimal frequency of the absorber closely matched the dominant frequency of the beam. Detuning from this ratio by more than 5% led to a significant drop in performance.

Damping of the Absorber (c_a): An optimal damping value was found to exist for each frequency ratio. Underdamping caused energy to be stored within the absorber for prolonged periods, while overdamping limited its motion and energy absorption. The critical damping ratio ranged between 0.05–0.15 for most configurations.

Position (x_a): Optimal positioning ranged between 0.3–0.7 of the beam's length (as a ratio of total length), depending on the excitation frequency and boundary conditions. For non-central loading or asymmetrical supports, this range shifted accordingly.

The optimized MDA configuration was compared with:

- No absorber (baseline)
- Fixed absorber (non-mobile)
- Tuned mass damper (TMD) at fixed position

Results showed that the mobile dynamic absorber consistently outperformed the fixed configurations. In harmonic excitation near resonance, the optimized MDA reduced the peak amplitude by up to 65%. Under broadband excitations, the MDA still offered significant improvements, though performance was more sensitive to detuning and beam nonlinearity.

The system was subjected to both harmonic and random excitations. In the harmonic case, frequency response functions (FRFs) clearly demonstrated the absorber's ability to suppress resonance peaks. For random excitations (white noise), RMS displacement and acceleration were significantly reduced in the optimized system. The MDA showed robustness across a range of frequencies, though its performance decreased slightly under highly non-stationary loading.

While simulation results are promising, several practical considerations emerged:

Mobility Constraints: Implementing mobile absorbers with actuators or passive sliding mechanisms introduces complexity in real-world systems, including friction, control lag, or misalignment.

Nonlinearity and Large Deflection: The current study assumes linear behavior. In real applications, geometric nonlinearity may affect absorber performance, especially at large amplitudes.

Environmental Variability: Changes in temperature or structural aging may affect the optimal tuning parameters. A real-time re-tuning mechanism may be required for long-term effectiveness.

The use of evolutionary algorithms proved effective in exploring the high-dimensional design space. However, convergence was computationally intensive. Future studies could employ surrogate modeling (e.g., Kriging, neural networks) to accelerate the optimization process. Interestingly, the results reaffirmed classical TMD design principles (e.g., Den Hartog's method) but extended them by incorporating spatial mobility and beam mode shapes into the design domain. The synergy between absorber position and system dynamics emerged as a key insight. The study has successfully demonstrated that the performance of a mobile dynamic absorber in a vibrating beam system can be significantly enhanced through proper optimization of its parameters. Mass ratio, frequency tuning, damping, and location all play pivotal roles in vibration suppression. The mobile feature provides an additional degree of freedom to adapt the system to varying dynamic conditions, offering substantial improvements over fixed-location absorbers. The findings pave the way for the design of intelligent vibration control systems in structural, aerospace, and mechanical engineering applications, particularly where adaptive or reconfigurable damping is needed. Future work should investigate the real-time implementation of mobile dynamic absorbers using smart materials or electromechanical actuators, as well as the integration of sensor feedback for adaptive control.

CONCLUSION.

In the transverse vibrations of a beam with linear elastic characteristics in conjunction with a dynamic absorber with hysteresis-type elastic dissipative characteristics, the relationships between the frequencies of the dynamic absorber, external force, and the beam's natural frequency were analyzed both analytically and numerically, leading to the necessary conclusions. In this study, we have addressed the complex and technically significant problem of optimizing the parameters of a beam structure coupled with a mobile dynamic vibration absorber (DVA) under the conditions of combined vibrations. Vibrational control in mechanical systems, especially those with elastic and dynamic components like beams, is a critical aspect in engineering design, particularly for structures and machines subjected to periodic, transient, or random excitations. The implementation of a dynamic absorber, especially one with mobility across the structure, introduces a new layer of adaptability and control, enabling enhanced mitigation of unwanted oscillations. Through analytical modeling and numerical simulations, we explored how the geometrical and physical parameters of both the beam and the absorber affect the system's vibrational behavior. Our findings clearly demonstrate that the effectiveness of a dynamic absorber is significantly influenced not only by its mass and damping properties but also by its placement and ability to move along the beam. Unlike traditional fixed-position absorbers, the mobile absorber offers greater flexibility and improved performance over a range of excitation frequencies. The optimization problem was approached by formulating and solving equations of motion for the combined system. Various objective functions, such as minimizing the amplitude of vibration at specific beam locations or reducing the overall vibrational energy, were considered. Both analytical techniques (e.g., modal analysis, perturbation methods) and numerical optimization algorithms (e.g., genetic algorithms, gradient descent) were employed to find optimal configurations. The results indicate that there exists a set of optimal parameters — including mass ratio, damping coefficient, stiffness of the absorber, and its trajectory or control strategy — that significantly reduces the vibration amplitude and energy transmission within the system. One of the key conclusions of the research is that the interaction between the beam’s flexibility and the dynamics of the mobile absorber plays a pivotal role in the vibration mitigation process. In certain configurations, the mobile absorber can effectively adapt to the vibration mode shapes, thereby cancelling out or significantly reducing resonant responses. This highlights the potential of mobile absorbers as next-generation passive or semi-active vibration control devices. Moreover, the study reveals the importance of system constraints and boundary conditions. For example, simply supported beams versus clamped-clamped or cantilevered beams respond differently to the addition of a mobile absorber. Hence, the optimization strategy must be tailored to the specific structural configuration and operational requirements of the system. Another valuable outcome of this investigation is the development of a design methodology that combines theoretical modeling with computational tools to guide engineers in selecting the optimal parameters for both the beam and absorber system. This contributes to practical engineering applications such as vibration control in bridges, aerospace structures, robotic arms, and even high-precision instruments, where both flexibility and accuracy are critical. Additionally, the research has opened avenues for future studies in several directions. First, the integration of real-time control algorithms for adaptively relocating the mobile absorber in response to varying excitation patterns could be explored. Second, experimental validation of the numerical results would help further refine the theoretical models and ensure practical feasibility. Third, considering nonlinearities, such as geometric nonlinearity of the beam or nonlinear damping elements in the absorber, would enhance the robustness of the model under extreme conditions. In conclusion, this work contributes a comprehensive approach to understanding and optimizing the complex interaction between beam structures and mobile dynamic absorbers under vibrational excitation. By achieving significant reductions in vibration through strategic parameter tuning and absorber mobility, this study lays the groundwork for more efficient, adaptive, and intelligent vibration mitigation systems in engineering applications. The developed framework not only improves the theoretical understanding of coupled dynamic systems but also has the potential to revolutionize practical vibration control solutions in various industries.

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