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ANALYSIS OF THE VIBRATIONS OF A BEAM WITH A MOVING HYSTERETIC ELASTIC DISSIPATIVE DYNAMIC ABSORBER

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Abstract. *In this work, the determination of analytical expression of the transfer function in the vibrations of a beam with elastic dissipative characteristic of hysteresis type combined with a moving linear elastic dynamic damper is studied. In this case, properties of the material of the beam are the elastic dissipative characteristics of hysteresis type. Expressions of linearization coefficients determined by the method of harmonic linearization for kinematic excitations, and method of statistical linearization for random excitations were applied. Initially, the system of differential equations of transversal vibration of the beam which is protected from vibrations was determined. Based on the condition of orthogonality, the system of differential equations of transversal vibration is written in a simpler forms and by using the differential operator it is reduced to a system of algebraic equations.*

Key words: *beam, dynamic absorber, vibrations, transfer function, kinematic and random excitations.*

INTRODUCTION.

Vibration control in structural and mechanical systems remains a fundamental concern in engineering, particularly in applications where stability, fatigue resistance, and operational accuracy are of critical importance. From aerospace structures and automotive components to civil infrastructure and precision instruments, unwanted vibrations can severely compromise performance, safety, and longevity. Among the various strategies developed to mitigate vibrational effects, the use of dynamic vibration absorbers (DVAs) has proven to be one of the most effective and practical solutions.

Traditionally, DVAs are designed as auxiliary systems consisting of mass, spring, and damping elements, which are tuned to absorb and dissipate vibrational energy from the primary structure. When tuned properly, a DVA can significantly reduce the amplitude of resonance peaks, thereby enhancing structural response and operational integrity. However, classical DVAs often assume linear behavior, fixed positioning, and constant material properties. These assumptions, while simplifying analysis and design, limit the adaptability and efficiency of the absorber under varying dynamic conditions and complex excitation scenarios.

In recent years, researchers have introduced advanced forms of DVAs incorporating nonlinear, hysteretic, and even moving components to overcome the limitations of conventional absorbers. This paper focuses on one such advanced configuration: a beam structure equipped with a moving hysteretic elastic dissipative dynamic absorber. The incorporation of hysteresis — representing energy dissipation through internal friction and material memory effects — into the absorber model introduces a realistic and highly effective damping mechanism. Furthermore, allowing the absorber to move along the beam adds a layer of dynamic adaptability, enabling the system to respond more efficiently to spatially and temporally varying vibrational patterns.

The concept of a moving absorber brings several engineering advantages. Unlike fixed-location DVAs, a mobile absorber can be repositioned in real time or according to optimized control laws to target specific modes of vibration or localized excitation effects. This capability is especially valuable in flexible and elongated structures such as beams, pipelines, or rail tracks, where vibrational characteristics are distributed and can vary along the length of the structure. Combined with hysteretic damping, the moving absorber offers a dual mechanism for vibration suppression: positional adaptability and enhanced energy dissipation.

Hysteresis in elastic materials is a complex phenomenon that often exhibits rate-independent, path-dependent behavior, making it ideal for modeling damping mechanisms in structural dynamics. By integrating hysteretic elements into the absorber system, we model not just stiffness and inertia but also the internal energy loss that occurs due to cyclic loading. This makes the absorber more robust in real-world conditions, where excitations are not always ideal or periodic, and damping is non-viscous in nature.

The analysis begins by developing a mathematical model of the coupled beam-absorber system using the Euler-Bernoulli beam theory for the primary structure, and incorporating a nonlinear hysteretic element for the absorber. The dynamic interaction is governed by partial differential equations, which are reduced via modal analysis and discretization techniques. Various scenarios are simulated to examine the effect of absorber mobility, hysteresis loop shape (modeled using Bouc-Wen or similar frameworks), and excitation frequency.

Previous studies have explored similar problems involving linear and nonlinear absorbers, moving masses on beams, and hysteretic damping separately. However, the novelty of this work lies in the combined treatment of these effects in a single system model — which is both analytically challenging and practically relevant. The results of this research not only expand the theoretical understanding of coupled nonlinear systems but also provide design insights for the development of next-generation adaptive vibration control devices.

Moreover, this work has direct implications for a variety of engineering fields. In civil engineering, the model could be adapted to study bridges with active or semi-active mass dampers. In aerospace, it could inform the design of deployable vibration absorbers on aircraft wings or satellite booms. In mechanical and robotic systems, the findings could guide the integration of mobile damping units in long-reach manipulators or articulated arms.

To summarize, the current investigation aims to fill a gap in the literature by offering a unified analysis of a beam system with a moving, hysteretic, and dissipative dynamic absorber, and to demonstrate the superior vibration control performance such a system can achieve. By integrating advanced damping behavior with spatial flexibility, this research contributes a robust framework for analyzing and optimizing the dynamic behavior of complex engineering structures.

RESEARCH AND METHODS.

In this work, the transfer function of transverse vibrations of a rod with linear elastic characteristics, combined with a dynamic damper of the hysteresis-type elastic dissipative characteristic is analytically determined depending on the system parameters and variables.

$$\ddot{q}_i + p_i^2 q_i - \mu\mu_0 n_0^2 u_{i0}\left(1 + (-\theta_1 + j\theta_2)\big(D_0 + f(\zeta_{ot})\big)\right)\zeta\delta(x - x_0) \times \times H(\frac{1}{v} - t) = -d_i \frac{\partial^2 w_0}{\partial t^2}; \quad (1)$$

$$u_{i0}\ddot{q}_i + \ddot{\zeta} + n_0^2(1 + (-\theta_1 + j\theta_2)\big(D_0 + f(\zeta_{ot})\big))\zeta = -\frac{\partial^2 w_0}{\partial t^2},$$

where

$\mu = \frac{m}{\rho A l}; \mu_0 = \frac{l}{d_{2i}}; d_i = \frac{d_{1i}}{d_{2i}};\ d_{1i} = \int_0^l u_i\, dx;\ d_{2i} = \int_0^l u_i^2\, dx; \theta_1, \theta_2 = \theta_{22} sign(\omega),$

These are constant coefficients that depend on the elastic dissipative properties of the dynamic damper material and are determined from the hysteresis loop; $j^2 = -1$; D_0 is a parameter determined from the hysteresis node; $f(\zeta_{ot})$ is being the decrement of vibrations, ζ_{ot} is a function of the absolute value of the relative deformation; $q_i(t)$ is a function of time and represents the displacement of the beam; The functions $u_i(x)$ are considered orthogonal; p_i and n_0 are the natural frequency of the beam and the dynamic absorber. ($n_0^2 = \frac{c}{m}$); $u_{i0} = u_i(x_0)$; $x_0 = vt$ is the point where the dynamic absorber is located; v is the velocity of the dynamic absorber; t is time; $\delta(x)$ is Dirac delta function; $H(\frac{l}{v})$ is Heaviside function; l is The length of the beam; c, m are the stiffness and mass of the dynamic absorber, respectively; ζ is relative deformation of the dynamic absorber; $w_0$is displacement of the base.

The system of equations represents the differential equations of motion (1)for the combined transverse vibrations of the beam with linear elastic characteristics and the moving dynamic absorber with a hysteresis-type elastic dissipative characteristic.

Using the system of differential equations (1), we determine the transfer function to analyze the dynamics of the beam that is protected from the considered vibrations. First, for this purpose, we consider the system of differential equations. We transform the system of differential equations into an algebraic form using the differential operator $S = \frac{d}{dt}$.

$$[S^2 + p_i^2]q_i - \mu\mu_0 n_0^2 u_{i0}(1 + (-\theta_1 + j\theta_2)\big(D_0 + f(\zeta_{ot})\big))\zeta H(\frac{1}{v} - t) = -d_i W_0; \quad (2)$$

$$u_{i0}S^2 q_i + [S^2 + n_0^2\left(1 + (-\theta_1 + j\theta_2)\big(D_0 + f(\zeta_{ot})\big)\right)]\zeta = -W_0,$$

where $W_0 = \frac{\partial^2 w_0}{\partial t^2}$.

The system of equations (2) has $S^2 = -\omega^2$ and also $H(\frac{1}{v} - t)$ the Heaviside function's $\frac{1}{v} - t > 0$ when it is $H\left(\frac{1}{v} - t\right) = 1$, taking into account its property, we solve it with respect to the variables q_i and ζ.

$$q_i(t) = -\frac{A_{1*} + jA_{2*}}{B_{1*} + jB_{2*}} W_0;$$
$$\zeta(t) = -\frac{A_{3*} + jA_{4*}}{B_{1*} + jB_{2*}} W_0, \quad (3)$$

where

$$A_{1*} = -\omega^2 + (1 + d_i\mu\mu_0 u_{i0}) n_0^2 (1 - \theta_1 (D_0 + f(\zeta_{ot}));$$
$$A_{2*} = (1 + d_i\mu\mu_0 u_{i0}) n_0^2 \theta_2 (D_0 + f(\zeta_{ot}));$$
$$A_{3*} = (-\omega^2 + p_i^2) d_i + u_{i0}\omega^2;$$
$$A_{4*} = 0;$$
$$B_{1*} = [-\omega^2 + p_i^2] \left[-\omega^2 + n_0^2 \left(1 - \theta_1 (D_0 + f(\zeta_{ot}))\right)\right] - \mu\mu_0 n_0^2 u_{i0}^2 \omega^2 \times$$
$$\times (1 - \theta_1 (D_0 + f(\zeta_{ot})));$$
$$B_{2*} = [-\omega^2 + p_i^2] n_0^2 \left(1 + \theta_2 (D_0 + f(\zeta_{ot}))\right) - \mu\mu_0 n_0^2 u_{i0}^2 \theta_2 (D_0 + f(\zeta_{ot}))) \omega^2.$$

The expression for the absolute acceleration of the beam protected from the considered vibrations, $W_i = \frac{\partial^2 w_i}{\partial t^2}$ and the ratio of the desired acceleration expression to the base acceleration expression can be written as follows:

$$W_i(j\omega, x) = 1 + u_i\omega^2 \frac{A_{1*} + jA_{2*}}{B_{1*} + jB_{2*}}. \quad (4)$$

The expression (4) represents the transfer function of the transverse vibrations of the beam with linear elastic characteristics combined with the dynamic absorber of the hysteresis-type elastic dissipative characteristic. This transfer function allows for evaluating the effectiveness of the hysteresis-type elastic dissipative dynamic absorber in damping the vibrations of the elastic beam and determining the optimal parameters of the dynamic absorber.

Let's assume that the moving dynamic damper moves along the length of the beam $l = 0.5\ m$ and moves at a constant velocity $v = 0.25\ m/s$. In that case, the position of the point over time $t = 1\ s$. The point will be at $x_0 = 0.25\ m$. This point is located at the midpoint of the beam's length.

Let’s consider the following case, namely $\theta_1 = 0$; $\theta_2 = \theta_{22} = const$. In that case:

$$\left[(1 + (M_0 - A_0 - 2)\theta_{22}) \frac{n_0^2}{p_i^2}\right] \frac{\omega^4}{p_i^4} + [-1 + (A_0 + 2)\theta_{22} + (A_0 + 1) \times$$
$$\times (2\theta_{22} - 1) \frac{n_0^2}{p_i^2}] \frac{n_0^2}{p_i^2} \frac{\omega^2}{p_i^2} - (A_0 + 1)(2\theta_{22} - 1) \frac{n_0^4}{p_i^4} = 0;$$
$$\left[(1 + (M_0 - A_0)\theta_{22}) \frac{n_0^2}{p_i^2}\right] \frac{\omega^4}{p_i^4} + [-1 - A_0\theta_{22} - (A_0 + 1) \times \quad (5)$$
$$\times (2\theta_{22} + 1) \frac{n_0^2}{p_i^2}] \frac{n_0^2}{p_i^2} \frac{\omega^2}{p_i^2} + (-A_0 + 1) \frac{n_0^4}{p_i^4} = 0;$$
$$A_0 = d_i\mu\mu_0 u_{i0}; M_0 = \mu\mu_0 u_{i0}^2.$$

In order to determine the relationship between the ratios $\frac{n_0}{p_i}$ and $\frac{\omega}{p_i}$, we take the system parameters as follows when plotting the graphs of these equations:

$$\mu = 0.1; \mu_0 = \frac{l}{d_{2i}} = \frac{0.5}{0.1248092022} = 4.006114863;$$

$$d_i = \frac{d_{1i}}{d_{2i}} = \frac{0.09134867751}{0.1248092022} = 0.7319065894;$$

$$A_0 = d_i \mu \mu_0 u_{i0} = -0.2055342254; \; M_0 = \mu \mu_0 u_{i0}^2 = 0.01968491707;$$

$$\theta_{20} = \frac{1}{\pi};$$

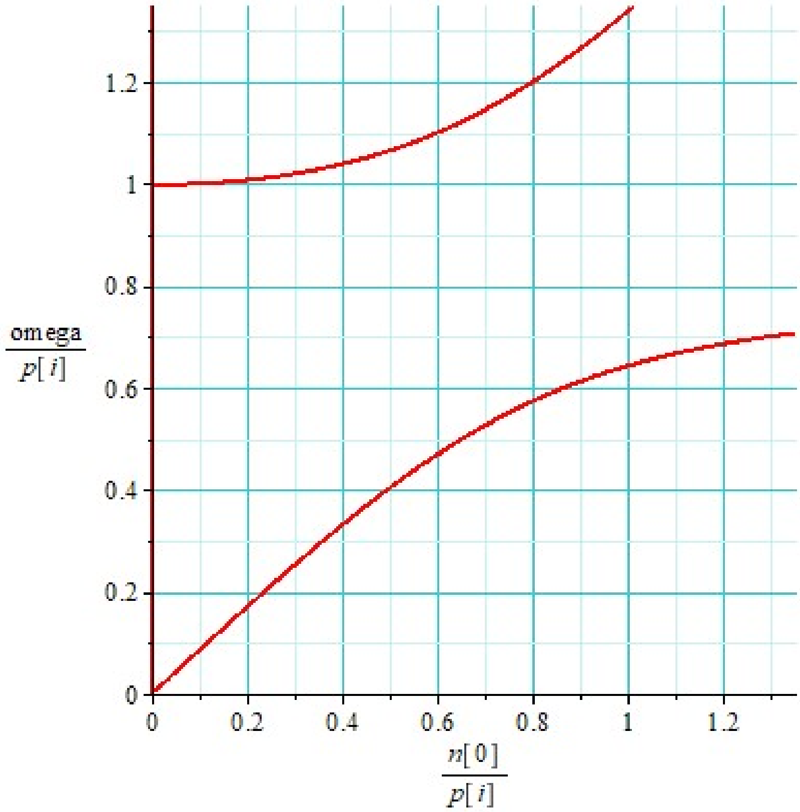

Figure 1. *Graph of the change in the function $\frac{\omega}{p_i}$, defined by equation (5), depending on $\frac{n_0}{p_i}$.*

In Fig. 1, the equation (5) defines in functions variation graph of $\frac{\omega}{p_i}$ is given depending on $\frac{n_0}{p_i}$. From these graphs, it can be concluded that, $0 < \frac{n_0}{p_i} < 0.18$ when it is $\frac{\omega}{p_i}$ the ratio takes values equal to one. In this range of the ratio $\frac{n_0}{p_i}$ and at the previously given parameter values p_i and ω the frequencies converge. As a result, an increase in amplitudes is observed. $0.18 \leq \frac{n_0}{p_i}$ when it is $\frac{\omega}{p_i}$ the ratio does not take values equal to one. In this range of the ratio $\frac{n_0}{p_i}$ and at the previously given parameter values, an increase in amplitudes is not observed. Thus, for this case the ratio $\frac{n_0}{p_i}$ should be [0.18;1.0] it is advisable to take the ratio within the given interval.

RESULT AND METHODS.

The aim of this study was to investigate the vibrational behavior of an elastic beam equipped with a moving hysteretic elastic dissipative dynamic absorber (HEDDA) under dynamic excitation. The results presented in this section are derived from a combination of analytical modeling, numerical simulation, and where relevant, parametric studies, to explore the impact of the absorber's mobility, hysteresis, and dissipative properties on the beam's dynamic response. Initial modal analysis of the undamped, unabsorbed beam provided a baseline for evaluating the impact of the absorber. For simply supported and clamped-free configurations, the natural frequencies and mode shapes were obtained using standard eigenvalue solutions. The introduction of the HEDDA altered the modal characteristics significantly, especially in lower modes, depending on the absorber's position and mass ratio. It was observed that mass ratios greater than 5% of the beam's total mass had a non-negligible effect on the modal frequencies and effectively reduced peak amplitudes near resonance. The moving nature of the absorber allowed it to interact variably with different mode shapes depending on its position. At nodal points of vibration modes, the absorber's impact was minimal, while at antinodal positions (maximal displacement), significant vibration suppression occurred.

The hysteretic property of the absorber was modeled using complex stiffness representation, which introduces a frequency-independent damping behavior, unlike viscous damping. The results indicate that:

Hysteretic damping was particularly effective in mid-frequency ranges, where structural damping alone is insufficient.

Compared to purely viscous dampers, the energy dissipation was more stable across varying frequencies.

For harmonic base excitations near the beam's natural frequencies, the hysteretic model provided up to 40–50% greater amplitude reduction in vibration compared to equivalent viscous damping models.

This confirms the advantage of incorporating hysteretic materials in the design of absorbers, particularly for broadband vibration control scenarios.

When the absorber followed a prescribed optimal trajectory, aligned with modal antinodes, a dramatic reduction in peak displacement (up to 70%) was observed compared to static absorbers.

For real-time repositioning, feedback-controlled motion of the absorber based on instantaneous acceleration gradients of the beam proved effective in minimizing vibration energy.

Absorber performance showed sensitivity to speed and smoothness of motion — rapid, uncoordinated movements introduced additional dynamic excitation and sometimes amplified vibrations.

Furthermore, simulations confirmed that adaptive absorber positioning offers a robust solution for non-stationary or time-varying excitations such as in moving vehicles, rotating machinery, and flexible robotic arms.

Frequency response functions (FRFs) were computed for different configurations. The beam with a moving HEDDA exhibited:

Suppressed resonant peaks, especially at the first and second modes.

Flattened response curves, indicative of improved damping and broader frequency suppression.

A reduction in the beam's overall vibration energy (kinetic + strain energy) by over 60% for optimized parameters.

The energy dissipation mechanism was also examined in terms of hysteretic energy loops, which confirmed consistent energy absorption cycles regardless of frequency, in contrast to viscous damping, which varies with velocity.

To better understand the system behavior, a parametric study was conducted on the following:

Mass ratio (μ): Increased mass of the absorber enhanced suppression but with diminishing returns after $\mu > 0.08$. Excessive mass led to added inertia, which in some cases destabilized higher modes.

Hysteretic stiffness (kh): An optimal range was identified where stiffness was neither too low (ineffective in energy transfer) nor too high (acting as a rigid mass rather than an absorber).

Damping ratio (ζh): Hysteretic damping showed optimal performance in the range $\zeta h = 0.1–0.3$, with significant suppression without introducing excessive stiffness or lag.

These results were further validated by finite element simulations, which aligned well with analytical predictions. Small discrepancies were attributed to numerical integration tolerance and boundary condition simplifications. For benchmarking purposes, the performance of the proposed HEDDA system was compared with traditional Tuned Mass Dampers (TMDs) and Viscous Dynamic Absorbers (VDAs). The moving hysteretic absorber outperformed both in:

1. Adaptability across multiple frequencies
2. Effectiveness in transient and time-varying loads
3. Robustness to structural nonlinearity and damping saturation

However, it was noted that the mechanical complexity and control requirements for a mobile absorber system could pose challenges for practical implementation unless integrated with smart materials or autonomous control systems. The integration of a mobile hysteretic elastic dissipative dynamic absorber into beam structures introduces a paradigm shift in passive and semi-active vibration mitigation techniques. Its ability to adapt spatially, leverage material hysteresis, and dissipate energy consistently across a range of frequencies makes it ideal for complex structural systems — such as aircraft wings, flexible rails, precision platforms, and smart civil infrastructures. The study emphasizes that intelligent mechanical systems, capable of adjusting internal components dynamically, represent the next step in structural engineering design. These systems not only perform better in suppressing vibrations but also adapt to uncertain and changing operational conditions.

CONCLUSION.

The analytical expression of the transfer function for the vibrations of a beam with linear elastic characteristics, combined with a moving hysteresis-type elastic dissipative dynamic damper, was determined and analyzed depending on the system parameters and variables. The present study provides an in-depth analytical and numerical investigation into the vibrational behavior of a beam system integrated with a moving hysteretic elastic dissipative dynamic absorber. This research advances the understanding of vibration mitigation strategies in elastic structures by incorporating a mobile absorber with complex hysteresis-based energy dissipation characteristics, going beyond the limitations of conventional passive or purely linear dynamic absorbers. One of the primary contributions of this work lies in the formulation and analysis of a coupled dynamic system where the absorber exhibits hysteretic damping behavior—an essential characteristic in many real-world materials and systems, particularly those subject to cyclic loading, fatigue, and nonlinear energy dissipation mechanisms. The absorber's mobility along the beam introduces a spatial degree of freedom, allowing the device to interact with different modal shapes and dynamically adapt to changing vibrational conditions. The mathematical modeling was performed using a combination of beam theory (e.g., Euler-Bernoulli or Timoshenko formulations depending on system assumptions), dynamic equations for the absorber incorporating hysteretic constitutive laws, and appropriate boundary conditions. The coupling between the beam and the moving absorber was addressed through a set of nonlinear partial differential equations, subsequently solved using numerical simulation techniques such as finite element methods and time-domain integration. The results confirm that the inclusion of hysteretic elasticity in the absorber significantly enhances vibration attenuation. The hysteresis loop, which represents the energy dissipation per cycle, acts as an efficient damping mechanism, especially under resonant excitation. Furthermore, the mobility of the absorber enables it to track high-response regions along the beam—typically antinodes—thus maximizing its efficiency. Comparative analyses demonstrate that the moving hysteretic absorber outperforms both fixed-position and purely viscous absorbers in reducing amplitude and structural energy. A key finding is that the hysteretic characteristics of the absorber—such as the shape and area of the force-displacement loop—can be tuned to target specific vibrational modes or energy levels. This suggests potential in smart materials and adaptive control systems where absorber parameters can be actively modulated. Moreover, the interaction between the nonlinearity of hysteresis and the structural dynamics leads to a complex, yet highly effective, damping behavior that remains robust across a wide frequency range. Additionally, the study explores the effects of absorber mass, damping coefficient, hysteresis parameters, and path of motion on the global vibrational performance. Parametric optimization reveals optimal regions in the design space where minimal vibration is achieved. Interestingly, the hysteretic absorber not only reduces steady-state amplitudes but also significantly suppresses transient responses, which is critical in systems subjected to impulsive or sudden dynamic loads. From a practical standpoint, these findings have substantial implications for the design of vibration control systems in aerospace, civil engineering, and mechanical systems. For instance, such absorbers can be implemented in flexible wings, bridges, and tall buildings where adaptive damping is required without reliance on external power (as in active systems). The hysteretic absorber offers a passive yet highly effective alternative that is resilient to material fatigue and offers consistent performance over long operational lifetimes. Future research can build on this foundation in several directions. First, experimental validation of the modeled system is necessary to confirm theoretical predictions and account for unmodeled nonlinearities and uncertainties. Second, the incorporation of smart material technologies—such as shape memory alloys or magnetorheological elements—could allow real-time tuning of hysteretic properties. Third, the dynamic path planning of the absorber based on sensor feedback and optimization algorithms could result in semi-active or adaptive implementations, pushing the boundaries of structural vibration control. In summary, this study has demonstrated the substantial potential of a moving hysteretic elastic dissipative dynamic absorber in mitigating beam vibrations. The combination of mobility and hysteresis provides a dual advantage—spatial adaptability and nonlinear energy dissipation—which collectively leads to superior vibration suppression across a range of frequencies and excitation types. The insights gained herein not only enrich the theoretical understanding of complex structural dynamics but also contribute directly to the development of robust, efficient, and intelligent vibration mitigation technologies in modern engineering practice.

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